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EXPONENTIAL CONVEXITY OF THE FAVARD'S INEQUALITY AND RELATED RESULTS*

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A b s t r a c t: In this paper we prove positive semi-definiteness of matrices generated by differences deduced from unweighted and weighted Favard's inequality. This implies a surprising property of exponential convexity of this differences which allows us to deduce Gram's, Lyapunov's and Dresher's types of inequalities for this differences.

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1. INTRODUCTION AND PRELIMINARIES

Let f and p be two positive measurable real valued functions defined on $(a,b) \subseteq \mathbb{R}$ with $\int_a^b p(x)dx = 1$. From theory of convex functions (cf [1, 2, 6]), the well-known Jensen's inequality gives that for $t < 0$ or $t > 1$,

$$\int_a^b p(x) f^t(x)dx \geq \left(\int_a^b p(x) f(x)dx\right)^t, \tag{1}$$

and reverse inequality holds for $0 < t < 1$.

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Let us write the well-known Favard's inequality:

Theorem 1.1. *Let f be a concave non-negative function on* $[a, b] \subset \mathbb{R}$. If $q > 1$, *then*

$$\frac{2^q}{q+1}\left(\frac{1}{b-a}\int_a^b f(x)dx\right)^q \geq \frac{1}{b-a}\int_a^b f^q(x)dx. \tag{2}$$

If $0 < q < 1$, *then the reverse inequality holds in* (2).

Let us note that Theorem 1.1 can be obtained from the following result, also obtained by Favard (cf. [7, p. 212]).

Theorem 1.2. *Let f be a non-negative continuous concave function on [a; b], not identically zero, and ϕ be a convex function on* $[0, 2\tilde{f}]$, *were*

$$\tilde{f} = \frac{1}{b-a}\int_a^b f(x)dx. \tag{3}$$

Then

$$\frac{1}{2\tilde{f}}\int_0^{2\tilde{f}} \phi(y)dy \geq \frac{1}{b-a}\int_a^b \phi(f(x))dx. \tag{4}$$

Karlin and Studden (cf. [3, p. 412]) gave a more general inequality as follows:

Theorem 1.3. *Let f be a non-negative continuous concave function on* $[a, b]$, *not identically zero, $\tilde{f}$ be defined in (3) and let ϕ be a convex function on* $[c, 2\tilde{f} - c]$ *where c satisfies $0 < c \leq f_{min}$ (where f_{min} is the minimum of f) and $\tilde{f}$ is defined in (3). Then*

$$\frac{1}{2\tilde{f}-2c}\int_c^{2\tilde{f}-c} \phi(y)dy \geq \frac{1}{b-a}\int_a^b \phi(f(x))dx. \tag{5}$$

For $\phi(y) = y^p,\ p > 1,$ we can get from Theorem 1.3:

Theorem 1.4. *Let f be continuous concave function such that $0 < c \le f_{\min}$, $\tilde{f}$ is defined in (3). If p > 1, then*

$$\frac{1}{(2\tilde{f}-2c)(p+1)}\left(\left(2\tilde{f}-c\right)^{p+1}-c^{p+1}\right) \ge \frac{1}{b-a}\int_a^b f^p(x)dx. \tag{6}$$

If $0 < p < 1$, *then the reverse inequality holds in* (6).

Definnition 1.5. A function $h:(a,b)\to\mathbb{R}$ is exponentially convex function if it is continuous and

$$\sum_{i,j=1}^{n} \xi_i \xi_j h(x_i + x_j) \ge 0$$

for all $n\in\mathbb{N}$ and all choices $\xi_i \in\mathbb{R}$ and $x_i \in (a,b)$, i = 1,…, n such that $x_i + x_j \in (a,b)$, $1 \le i, j \le n$.

The following proposition is given in [4]:

Proposition 1.6. *Let* $h:(a,b)\to\mathbb{R}$. *The following propositions are equivalent.*

(i) *h is exponentially convex.*

(ii) *h is continuous and*

$$\sum_{i,j=1}^{n} \xi_i \xi_j h\left(\frac{x_i + x_j}{2}\right) \ge 0,$$

for every $n\in\mathbb{N}$, *every* $\xi_i \in\mathbb{R}$ *and every* $x_i, x_j \in (a,b)$, $1 \le i$, $j \le n$.

Corollary 1.7. *If h is exponentially convex then*

$$\det\left[h\left(\frac{x_i + x_j}{2}\right)\right]_{i,j=1}^{n} \ge 0,$$

for every $n\in\mathbb{N}$, *and every* $x_i \in (a,b)$, $i = 1, ..., n$.

Corollary 1.8. *If* $h:(a,b)\to\mathbb{R}^+$ *is exponentially convex function then h is a log-convex functions.*

With the help of following useful Lemma we prove our results:

Lemma 1.9. *Define the functions*

$$\varphi_s(x)=\begin{cases}\dfrac{x^s}{s(s-1)}, & s\neq 0,1;\\ -\log x, & s=0;\\ x\log x, & s=1.\end{cases}\tag{7}$$

Then $\varphi_s''(x)=x^{s-2}$, *that is,* $\varphi_s(x)$ *is convex for* $x>0$.

In second section we prove positive semi-definiteness of matrices generated by differences deduced from Favard's inequalities (2) and (6). This implies a surprising property of exponential convexity of this differences which allows us to deduce Gram's, Lyapunov's and Dresher's types of inequalities for this differences. Our results are extensions of results for log-convexity given in [5].

2. FAVARD'S INEQUALITY

Theorem 2.1. *Let f be a positive continuous concave function on* $[a,b]$, $\tilde{f}$ *be defined in (3) and*

$$\Omega_s(f):=\begin{cases}\dfrac{1}{s(s-1)}\left[\dfrac{2^s}{x+1}\left(\dfrac{1}{b-a}\int_a^b f(x)dx\right)^s-\dfrac{1}{b-a}\int_a^b f^s(x)dx\right], & s\neq 0,1,\\ 1-\log 2-\log\tilde{f}+\dfrac{1}{b-a}\int_a^b \log f(x)dx, & s=0,\\ \log 2\tilde{f}+\tilde{f}\log\tilde{f}-\dfrac{1}{2}\tilde{f}-\dfrac{1}{b-a}\int_a^b f(x)\log f(x)dx, & s=1.\end{cases}\tag{8}$$

Then the following statements are valid:

(a) *For every* $n \in \mathbb{N}$ *and* $s_1,\ldots,s_n \in \mathbb{R}$, *the matrix* $\left[\Omega_{\frac{s_i+s_j}{2}}\right]_{i,j=1}^{n}$ *is a positive semi-definite matrix. Particularly*

$$\det\left[\Omega_{\frac{s_i+s_j}{2}}\right]_{i,j=1}^{k} \geq 0 \tag{9}$$

for $k = 1, \ldots, n$.

(b) *The function* $s \to \Omega_s$ *is exponentially convex.*

(c) *The function* $s \to \Omega_s$ *is a log-convex on* $\mathbb{R}$ *and the following inequality holds for* $-\infty < r < s < t < \infty$:

$$\Omega_s^{t-r} \leq \Omega_r^{t-s}\Omega_t^{s-r}. \tag{10}$$

Proof. (a) Consider the function

$$\phi(x) = \sum_{i,j}^{k} u_i u_j \varphi_{s_{ij}}(x)$$

for $k = 1, \ldots, n$, $x > 0$, $u_i \in \mathbb{R}$, $s_{ij} \in \mathbb{R}$, where $s_{ij} = \dfrac{s_i + s_j}{2}$ and $\varphi_{s_{ij}}$ is defined in (7). Set

$$\phi(x) = \sum_{i,j}^{k} u_i u_j \varphi_{s_{ij}}(x)$$

We have

$$\phi''(x) = \sum_{i,j}^{k} u_i u_j x^{s_{ij}-2}$$

$$= \left(\sum_{i}^{k} u_i x^{\frac{s_i}{2}-1}\right)^2 \geq 0, \quad x \geq 0.$$

This shows that ϕ is a convex function for $x \geq 0$.

Using Theorem 1.2,

$$\frac{1}{2\tilde{f}}\int_0^{2\tilde{f}} \phi(y)dy \geq \frac{1}{b-a}\int_a^b \phi(f(x))dx,$$

we have

$$\frac{1}{2\tilde{f}}\int_0^{2\tilde{f}} \left(\sum_{i,j}^{k} u_i u_j \varphi_{s_{i,j}}(y)\right)dy$$

$$\geq \frac{1}{b-a}\int_a^b \left(\sum_{i,j}^{k} u_i u_j \varphi_{s_{i,j}}(f(x))\right)dx,$$

or equivalently

$$\sum_{i,j}^{k} u_i u_j \,\Omega_{s_{i,j}} \geq 0.$$

From last inequality, it follows that the matrix $\left[\Omega_{\frac{s_i+s_j}{2}}\right]_{i,j=1}^{k}$ is a positive semi-definite matrix, that is, (9) is valid.

(b) Note that Ω_s is continuous for $s \in \mathbb{R}$ since

$$\lim_{s\to 0}\Omega_s = \Omega_0 \quad \text{and} \quad \lim_{s\to 1}\Omega_s = \Omega_1.$$

Then by using Proposition 1.6, we get exponentially convexity of the function $s \to \Omega_s$.

(c) A simple consequence of Corollary 1.8 is that Ω_s is log-convex, then by definnition

$$\log\Omega_s^{t-r} \leq \log\Omega_r^{t-s} + \log\Omega_t^{s-r}.$$

which is equivalent to (10). □

Remark 2.2. Theorem 2.1 (c) was proved in [5].

Remark 2.3. The following result was also proved in [5] as a consequence of Theorem 2.1 (c).

Let *f*, $\Omega_s(f)$ be defined in Theorem 2.1 and *t*, *s*, *u*, *v* be real numbers such that $s \le u$, $t \le v$, $s \ne t$, $u \ne v$, we have

$$\left(\frac{\Omega_s(f)}{\Omega_s(f)}\right)^{\frac{1}{t-s}} \le \left(\frac{\Omega_v(f)}{\Omega_u(f)}\right)^{\frac{1}{v-u}}. \tag{11}$$

Namely, in (cf [6], p.2), we have the following result for convex function *f* with $x_1 \le y_1$, $x_2 \le y_2$, $x_1 \ne x_2$, $y_1 \ne y_2$,

$$\frac{f(x_2)-f(x_1)}{x_2-x_1} \le \frac{f(y_2)-f(y_1)}{y_2-y_1}. \tag{12}$$

Since by Theorem 2.1, $\Omega_s(f)$ is log-convex, we can set in (12): $f(x)=\log\Omega_x$ and $x_1 = s$, $x_2 = t$, $y_1 = u$, $y_2 = v$. We get

$$\frac{\log\Omega_t(f)-\log\Omega_s(f)}{t-s} \le \frac{\log\Omega_v(f)-\log\Omega_u(f)}{v-u},$$

$$\log\left(\frac{\Omega_t(f)}{\Omega_s(f)}\right)^{\frac{1}{t-s}} \le \log\left(\frac{\Omega_v(f)}{\Omega_u(f)}\right)^{\frac{1}{v-u}},$$

after applying exponential function, we get (11).

Theorem 2.4. *Let f be a continuous concave function on* $[a,\ b]$ *such that* $0 \le c \le f_{\min}$, $\tilde{f}$ *be defined in* (3) *and*

$$\Lambda_s(f) := \begin{cases} \dfrac{1}{s(s-1)}\left[\dfrac{(2\tilde{f}-c)^{s+1}}{(2\tilde{f}-2c)(s+1)} - \dfrac{c^{s+1}}{(2\tilde{f}-2c)(s+1)} - \dfrac{1}{b-a}\int_a^b f^s(x)dx\right], & s \neq 0,1, \\ \dfrac{1}{2\tilde{f}-2c}\left[2\tilde{f} + c\log c - 2c - \left(2\tilde{f}-c\right)\log\left(2\tilde{f}-c\right)\right] & \\ \qquad + \dfrac{1}{b-a}\int_a^b \log f(x)dx, & s = 0, \\ \dfrac{1}{2\bar{f}-2c}\left[\left(2\tilde{f}-c\right)^2 \log\left(2\tilde{f}-c\right) - 2\tilde{f}^2 + 2c\tilde{f} - c^2\log c + 2c\right] & \\ \qquad - \dfrac{1}{b-a}\int_a^b f(x)\log f(x)dx, & s = 1. \end{cases} \tag{13}$$

Then the following statements are valid:

(a) *For every* $n \in \mathbb{N}$ and $s_1, \ldots, s_n \in \mathbb{R}$, *the matrix* $\left[\Lambda_{\frac{s_i+s_j}{2}}\right]_{i,j=1}^n$ *is a positive semi-definite matrix. Particularly*

$$\det\left[\Lambda_{\frac{s_i+s_j}{2}}\right]_{i,j=1}^k \geq 0 \tag{14}$$

for $k = 1, \ldots, n$.

(b) *The function* $s \to \Lambda_s$, *is exponentially convex*.

(c) *The function* $s \to \Lambda_s$, *is a log-convex on* $\mathbb{R}$ *and the following inequality holds for* $-\infty < r < s < t < \infty$:

$$\Lambda_s^{t-r} \leq \Lambda_r^{t-s}\Lambda_t^{s-r}. \tag{15}$$

Proof. As in the proof of Theorem 2.1, we use Theorem 1.4 instead of Theorem 1.2. □

Remark 2.5. Theorem 2.4 (c) was proved in [5].

Remark 2.6. The following result was also proved in [5] as a consequence of Theorem 2.4 (c).

Let f, $\Lambda_s(f)$ be defined in Theorem 2.4 and t, s, u, v be real numbers such that $s \le u, t \le v, s \ne t, u \ne v,$ we have

$$\left(\frac{\Lambda_t(f)}{\Lambda_s(f)}\right)^{\frac{1}{t-s}} \le \left(\frac{\Lambda_v(f)}{\Lambda u(f)}\right)^{\frac{1}{v-u}}. \tag{16}$$

3. WEIGHTED FAVARD'S INEQUALITY

The weighted version of Favard's inequality was obtained by L. Maligranda, J. E. Pečarić and L. E. Persson in [6].

Theorem 3.1.

(1) *Let f be a positive increasing concave function on* [*a, b*]. *Assume that* ϕ *is a convex function on* [0, ∞), *where*

$$\tilde{f}_i = \frac{(b-a)\int_a^b f(t)\, w(t)dt}{2\int_a^b (t-a)\, w(t)dt}. \tag{17}$$

Then

$$\frac{1}{b-a}\int_a^b \phi(f(t))\, w(t)dt \le \int_0^1 \phi(2r\, \tilde{f}_i)\, w[a(1-r)+br]dr. \tag{18}$$

If f is an increasing convex function on [*a, b*] *and f*(*a*) = 0, *then the reverse inequality in* (18) *holds*.

(2) *Let f be a positive decreasing concave function on* [*a, b*]. *Assume that* ϕ *is a convex function on* [0, ∞), *where*

$$\tilde{f}_d = \frac{(b-a)\int_a^b f(t)\, w(t)dt}{2\int_a^b (b-t)\, w(t)dt}. \tag{19}$$

Then

$$\frac{1}{b-a}\int_a^b \phi(f(t))\, w(t)dt \le \int_0^1 \phi(2r\, \tilde{f}_d)\, w[ar+b(1-r)]dr\,. \tag{20}$$

If f is a decreasing convex function on [*a, b*] *and f*(*b*) = 0*, then the reverse inequality in* (20) *holds*.

Theorem 3.2.

(1) *Let f be a positive increasing concave function on* [*a, b*], $\tilde{f}_i$ *is defined in* (17) *and*

$$\Pi_s(f) := \int_0^1 \varphi_s(2r\, \tilde{f}_i)\, w[a(1-r)+br]dr - \frac{1}{b-a}\int_a^b \varphi_s(f(t)\, w(t)dt. \tag{21}$$

Then the following statements are valid:

(a) *For every* $n \in \mathbb{N}$ *and* $s_1,\ldots,s_n \in \mathbb{R}$, *the matrix* $\left[\Lambda_{\frac{s_i+s_j}{2}}\right]_{i,j=1}^{n}$ *is a positive semi-definite matrix. Paticularly*

$$\det\left[\Pi_{\frac{s_i+s_j}{2}}\right]_{i,j=1}^{k} \ge 0 \tag{22}$$

for k = 1, …, *n*.

(b) *The function* $s \to \Pi_s$ *is exponentially convex*.

(c) *The function* $s \to \Pi_s$ *is a log-convex on* $\mathbb{R}$ *and the following inequality holds for* $-\infty < r < s < t < \infty$:

$$\Pi_s^{t-r} \le \Pi_r^{t-s}\Pi_t^{s-r}\,. \tag{23}$$

(2) *Let f be an increasing convex function on* [*a, b*], *f*(*a*) = 0, $\tilde{\Pi}_s(f) := -\Pi_s(f)$. *Then the following statements are valid*:

(a) *For every* $n \in \mathbb{N}$ *and* $s_1, \ldots, s_n \in \mathbb{R}$, *the matrix* $\left[\tilde{\Pi}_{\frac{s_i+s_j}{2}}\right]_{i,j=1}^{n}$ *is a positive semi-definite matrix. Particularly*

$$\det\left[\tilde{\Pi}_{\frac{s_i+s_j}{2}}\right]_{i,j=1}^{k} \geq 0 \tag{24}$$

for $k = 1, \ldots, n$.

(b) *The function* $s \to \tilde{\Pi}_s$ *is exponentially convex.*

(c) *The function* $s \to \tilde{\Pi}_s$ *is a log-convex on* $\mathbb{R}$ *and the following inequality holds for* $-\infty < r < s < t < \infty$:

$$\tilde{\Pi}_s^{t-r} \leq \tilde{\Pi}_r^{t-s} \tilde{\Pi}_t^{s-r} \tag{25}$$

Proof. As in the proof of Theorem 2.1, we use Theorem 3.1(1) instead of Theorem 1.2. □

Remark 3.3. Theorem 3.2 (c) was proved in [5].

Remark 3.4. The following result was also proved in [5] as a consequence of Theorem 3.2 (c).

(1) Let f and $\Pi_s(f)$ be defined as in Theorem 3.2(1) and $t, s, u, v \geq 0$ be such that $s \leq u$, $t \leq v$, $s \neq t$, $u \neq v$. Then

$$\left(\frac{\Pi_t(f)}{\Pi_s(f)}\right)^{\frac{1}{t-s}} \leq \left(\frac{\Pi_v(f)}{\Pi_u(f)}\right)^{\frac{1}{v-u}}. \tag{26}$$

(2) Let f and $\tilde{\Pi}_s(f)$ be defined as in Theorem 3.2(2) and $t, s, u, v \geq 0$ such that $s \leq u$, $t \leq v$, $s \neq t$, $u \neq v$. Then

$$\left(\frac{\tilde{\Pi}_t(f)}{\tilde{\Pi}_s(f)}\right)^{\frac{1}{t-s}} \leq \left(\frac{\tilde{\Pi}_v(f)}{\tilde{\Pi}_u(f)}\right)^{\frac{1}{v-u}}. \tag{27}$$

Theorem 3.5.

(1) *Let f be a positive decreasing concave function on* $[a, b]$, $\tilde{f}_d$ *be defined as in* (19) *and*

$$\Gamma_s(f) := \int_0^1 \varphi_s(2r\tilde{f}_d) w[ar + b(1-r)]\, dr - \frac{1}{b-a}\int_a^b \varphi_s(f(t))\, w(t)\, dt. \tag{28}$$

Then the following statements are valid:

(a) *For every* $n \in \mathbb{N}$ *and* $s_1, \ldots, s_n \in \mathbb{R}$, *the matrix* $\left[\Gamma_{\frac{s_i+s_j}{2}}\right]_{i,j=1}^{n}$ *is a positive semi-definite matrix. Particularly*

$$\det\left[\Gamma_{\frac{s_i+s_j}{2}}\right]_{i,j=1}^{k} \geq 0 \tag{29}$$

for $k = 1, \ldots, n$.

(b) *The function* $s \to \Gamma_s$ *is exponentially convex.*

(c) *The function* $s \to \Gamma_s$ *is a log-convex on* $\mathbb{R}$ *and the following inequality holds for* $-\infty < r < s < t < \infty$:

$$\Gamma_s^{t-r} \leq \Gamma_r^{t-s}\Gamma_t^{s-r} \tag{30}$$

(2) *Let* f *be a decreasing convex function on* $[a, b]$, $f(b) = 0$, $\tilde{\Gamma}_s(f) := -\Gamma_s(f)$. *Then the following statements are valid*:

(a) *For every* $n \in \mathbb{N}$ *and* $s_1, \ldots, s_n \in \mathbb{R}$, *the matrix* $\left[\tilde{\Gamma}_{\frac{s_i+s_j}{2}}\right]_{i,j=1}^{n}$ *is a positive semi-definite matrix. Paticularly*

$$\det\left[\tilde{\Gamma}_{\frac{s_i+s_j}{2}}\right]_{i,j=1}^{k} \geq 0 \tag{31}$$

for $k = 1, \ldots, n$.

(b) *The function* $s \to \tilde{\Gamma}_s$ *is exponentially convex.*

(c) *The function* $s \to \tilde{\Gamma}_s$ *is a log-convex on* $\mathbb{R}$ *and the following inequality holds for* $-\infty < r < s < t < \infty$:

$$\tilde{\Gamma}_s^{t-r} \le \tilde{\Gamma}_r^{t-s} \tilde{\Gamma}_t^{s-r}. \tag{32}$$

Proof. As in the proof of Theorem 2.1, we use Theorem 3.1(2) instead of Theorem 1.2.

Remark 3.6. Theorem 3.5 (c) was proved in [5].

Remark 3.7. The following result was also proved in [5] as a consequence of Theorem 3.5 (c).

(1) Let *f* and $\Gamma_s(f)$ be defined as in Theorem 3.5(1) and *t, s, u,* $v \ge 0$ be such that $s \le u,\ t \le v,\ s \ne t,\ u \ne v$. Then

$$\left(\frac{\Gamma_t(f)}{\Gamma_s(f)}\right)^{\frac{1}{t-s}} \le \left(\frac{\Gamma_v(f)}{\Gamma_u(f)}\right)^{\frac{1}{v-u}}. \tag{33}$$

(2) Let *f* and e $\hat{\Gamma}_s(f)$ be defined as in Theorem 3.5(2) and *t*; *s*; *u*; $v \ge 0$ such that $s \le u,\ t \le v,\ s \ne t,\ u \ne v$. Then

$$\left(\frac{\tilde{\Gamma}_t(f)}{\tilde{\Gamma}_s(f)}\right)^{\frac{1}{t-s}} \le \left(\frac{\tilde{\Gamma}_v(f)}{\tilde{\Gamma}_u(f)}\right)^{\frac{1}{v-u}}. \tag{34}$$

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Р е з и м е

ЕКСПОНЕНЦИЈАЛНА КОНВЕКСНОСТ НА FAVARD-ОВОТО НЕРАВЕНСТВО И СРОДНИ РЕЗУЛТАТИ

Во оваа статија ние докажуваме позитивна полудефинираност на матрици генерирани од разлики изведени од нетежински и тежински Favard-ови неравенства. Ова имплицира едно интересно својство на експоненцијалната конвексност на овие разлики, кои ни даваат можност да ги изведеме Gram-овите, Lyapunov-ите и Dresher-овите типови неравенства за овие разлики.

Клучни зборови: Favard-ово неравенство; тежинско Favard-ово неравенство; позитивна полудефинитна матрица; експоненцијална конвексност; log-конвексност

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