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МАКЕДОНСКА АКАДЕМИЈА НА НАУКИТЕ И УМЕТНОСТИТЕ
MACEDONIAN ACADEMY OF SCIENCES AND ARTS

ОДДЕЛЕНИЕ ЗА ТЕХНИЧКИ НАУКИ
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**ON THE EVE OF THE GREAT JUBILEE – 50 YEARS
OF THE MACEDONIAN ACADEMY OF SCIENCES AND ARTS
1967 – 2017**

This year the Macedonian Academy of Sciences and Arts (MASA) marks and celebrates a great jubilee – 50 years of existence and work of our highest institution in the field of sciences and arts. Although on 22 February 2017 the 50th anniversary of the enactment of MASA in the Assembly of the Socialistic Republic of Macedonia was marked, and on October 10 it will be 50 years since the solemn establishment of MASA, we proudly emphasize that our roots, the roots of the Macedonian and Slavic cultural and spiritual continuity, are far back, in a time dimension which is measured in centuries. Because the mission of the Ss. Cyril and Methodius, the historical events that made Ohrid, with the famous Ohrid Literary School, already in the IX century to become the center of the Slavic educational and enlightening activity, which then spread throughout all Slavic countries, have fundamentally changed our contribution to the treasury of the European culture and civilization. And furthermore, centuries later, in the middle of the XIX century the Macedonian revival began, with a pleiad of our cultural and national activists. These processes at the beginning of the XX century resulted in the establishment of the Macedonian Scientific and Literary Fellowship in Saint Petersburg, led by Dimitrija Chupovski and Krste Petkov Misirkov, whose rich scientific, literary and cultural activities were a significant reflection of our spiritual continuity and identity, and an event that has marked the dawn of the Macedonian Academy of Sciences and Arts. This continuity will remain in the period between the two world wars, with a pleiad of artists in literature, art, music, philological, economic, legal and technical sciences. A few years after World War II, in 1949, in free Republic of Macedonia, the first state University of “Ss. Cyril and Methodius” was established, within which, in less than two decades, solid personnel resources were created which allowed rapid development of the higher education and scientific activity in our country. It was an event of great importance for the establishment of MASA as the highest institution in the field of sciences and arts.

This millennium pace and continuity in the development of art and scientific thought in our region is an indication and evidence that we are not a nation without its own roots, without its own history, without its own culture, and that the attempts to deny our identity, language, name, no matter where they come from, are residual of the Balkan anachronisms, and essentially speaking, they are absurd and retrograde.

Immediately after the establishment of MASA followed a period of rapid development, diversification and enrichment of its scientific and research activities and artistic work. Almost two decades after the establishment MASA entered the phase of its maturity and has grown and has affirmed as the fundament of the Macedonian science, language, culture and history and as one of the pillars and symbols of the statehood of the Republic of Macedonia.

Today, MASA, according to its integral concept, structure and function, has all the necessary attributes of a modern national academy of European type, and of course, performs the three basic functions typical of the European national academies: creating communication space for confrontation of different views and opinions on important issues in the field of sciences and arts, scientific and research work and advisory role.

The scientific and research activities and artistic work, in fact, constitute the core of the activity of MASA. The number of completed scientific and research projects and projects in the field of arts within MASA is impressive – around 600 projects in the past 50 years. Some of these projects are long-term and are mainly related to the strategic issues of specific national interest, and significant is the number of fundamental and applied research in all fields of science and art represented in the Academy. MASA members in their scientific research increasingly incorporate the international dimension in the work – in the recent years more than 60% of the scientific papers have been published in international journals, most of which have been published in journals with impact factor; 50% of the papers that have been published in proceedings of scientific and professional meetings are related to meetings held abroad, etc. In addition, the works of our renowned writers and poets, members of MASA, are translated into foreign languages, and their work has found its place in world anthologies. Our prominent painters and sculptors of the older and the younger generation have created and create masterworks that are regularly exhibited at home and abroad. It should be particularly noted that our two research centers – Research Center for Energy and Sustainable Development and the Research Centre for Genetic Engineering and Biotechnology “Georgi D. Efremov”, that have gained high reputation in the region and beyond, continue to successfully maintain the attained position. The work of the other research centers also enhances, including the newly established ones, which have begun to work on significant international scientific and research projects.

In its half-century of existence and work MASA developed a rich publishing activity. Since its establishment until today around 700 titles have been published – monographs, results of scientific projects, proceedings from scientific meetings, music releases, facsimile and jubilee publications, joint publications with other academies and scientific institutions, publications of

solemn meetings, special issues of the departments of MASA etc. A special contribution to the publishing activities of MASA provides the “Trifun Kostovski” Foundation that has been existing and working for 18 years.

MASA proactively follows the changes and the new trends in the scope of the advisory function of the modern European national economies, and in that context the obligations arising from the project SAPEA - Science Advice for Policy by European Academies, initiated by the European Commission in order to intensify the cooperation of the European academies within their advisory role. Through the publication of the results of our scientific and research work, their presentation to the wider scientific and professional public in the country, to the government officials, etc., MASA participates in the policy-making in the field of sciences and arts and in the overall development of the country. The maintenance of the independence of MASA in carrying out the advisory role is our highest priority and principle.

In the recent years MASA has developed extensive international cooperation that contributes to the affirmation of the Macedonian scientific and artistic work and to the increasing of the reputation of MASA and of the Republic of Macedonia in international scale. Today, our Academy cooperates with more than 30 foreign academies and scientific societies and is a member of 7 international associations of academies. In the recent years the cooperation with the academies from the neighboring countries has been intensified, as well as with the Leibniz Society of Sciences from Berlin, and also, within the so-called Berlin process (Joint Science Conference of Western Balkans Process / Berlin Process) the cooperation with the German National Academy of Sciences – Leopoldina, with the French Academy of Sciences, the academies of Southeast Europe and others.

Due to the results achieved in its work, MASA and its members have won a number of high national and international awards. In the past 50 years, MASA has won around 90 awards and recognitions – charters, plaques, certificates of appreciation, medals and decorations from national and international scientific, educational, artistic and other institutions. Particularly, it should be noted that MASA has been awarded with the high decoration Order of the Republic of Macedonia for the contribution to the development of the scientific and research activity and artistic creativity of importance to the development and affirmation of the Macedonian science and state, which is awarded by the President of the Republic of Macedonia, as well as the prestigious Samuel Mitja Rapoport award of the Leibniz Society of Sciences from Berlin, which, for the first time, has been awarded to MASA. Today, 22 members of MASA have the status of foreign, corresponding and honorary members, as well as holders of honorary PhDs at around 60 foreign academies, scientific societies and universities.

The developmental trajectory of MASA unambiguously confirms that the Academy, in its 50 years of existence and work, faced with periods of heights, but also periods of descents and turbulences that are most directly linked to the situation in the Macedonian society, i.e. with crisis periods of different nature – the dissolution of the former common state (SFR Yugoslavia), problems with the recognition of the international status of the country after its independence, the embargoes and the blockades of the country in the early transition years, the internal conflict in 2001 and the political crisis in the last two-three years. In such crises and tense periods the criticism for the Academy grew – that MASA is an institution closed in itself, that MASA stays away from the current issues and developments in the country, and so on. On the one hand, it is a result of the insufficient understanding of the social role of the Academy – MASA is the highest scientific institution, where hasty reactions of columnist ‘type’, with daily political features are not characteristic. On the contrary, MASA uses facts and arguments. The basic activity of MASA, the results achieved in the scientific research and the artistic work is our identification within the national and international professional and scientific community, and beyond, within our society. On the other hand, this criticism and perception of MASA has a real basis in the fact that MASA, as opposed to the huge opus of implemented scientific and research and artistic projects still insufficiently affirms the results of its scientific and artistic production to the public. It is our weakness that we must overcome in the future. Of course, we cannot and must not “turn a blind eye” to the other weaknesses and omissions which, at least from time to time, we have faced with over the past 50 years and which we will face with in the future – insufficient scientific criticism of the events in the field of sciences and arts, insufficient resistance to political influence etc. On the contrary, in the future, we will have to clearly identify the weaknesses and the oversights in our work and to find out the right approaches to overcome them.

Today we live in a world of great science. The strong development of sciences, the new technological model based on information and communication technologies, the new wave of entrepreneurial restructuring of economies and societies, the globalization of the world economic activity, opened new perspectives to the economic growth and the development of individual countries and of the world economy as a whole. However, these processes, by their nature, are contradictory. The latest global financial and economic crisis of 2007-2009 revealed the contradictions of the globalization and the discontent of the people from it – the uneven distribution of wealth and power among individual countries, destruction of the resources and the environment worldwide, exhaustion of power of the existing technology and

development models. These processes resulted in other problems – refugee and migration crises, strengthening of the regional and national protectionism despite the efforts to liberalize the international trade, fencing of the countries with walls at the beginning of the new millennium, changes in the economic and technological power and of the geo-strategic position and importance of entire regions and continents, etc. Nevertheless, one thing is a fact – societies that aspire to grow into societies and knowledge-based economies more easily deal with all the above mentioned problems, challenges and risks of the modern world. Of course, moving towards a development knowledge-based model assumes large investments of resources in education, science, research and development and in culture, simultaneously accompanied by well-conceived and devised strategies on development of these crucial areas of the human spirit and civilizational endurance. Hence, this fact, undoubtedly, emphasizes the special significance of the national academies of sciences and arts in achieving this objective.

In the recent years the Republic of Macedonia has been facing with the most difficult political and social crisis in the period after its independence. We are facing a crisis of the institutions, breach of the principles of the rule of law, the phenomenon of “captured state”, a decline in the process of democratization of the society and falling behind on the road to the Euro-Atlantic integration processes. The problems that are now in the focus of our reality will require major reforms, much knowledge, energy and political will to overcome them. In this sense, and in this context, the role of MASA and of the overall scientific potential of the country in overcoming the crisis is also particularly important.

The above summarized evaluations and considerations about the development of MASA in the past 50 years, about the achievements in the realization of its basic activity, about the problems it faced and faces with, about the major challenges arising from the new age and which are determined with the changes in the international and national environment, they alone define the main priorities of our Academy in the forthcoming period:

- Our long-term goals are contained in the mission and vision of MASA as the highest institution in the field of sciences and arts. The mission of MASA is through the development of the basic functions that are characteristic for all modern national academies of European type, to give its full contribution to the inclusion of the Macedonian science and art in the modern European and world trends, and our vision is the Republic of Macedonia to become an advanced society based on science and knowledge;
- In the forthcoming years the focus of the scientific and research activity and artistic work of MASA, in cooperation with the other scientific and research institutions in the country and with government experts, will be particularly

focused on the elaboration of issues and topics that are most directly related to the sources of the current political and social crisis in the country in order to offer possible solutions, approaches and policies to overcome it;

- The issues related to the Euro-Atlantic integration processes of the Republic of Macedonia, their continuous and persistent scientific monitoring and elaboration and active participation of MASA members in the preparation for the accession negotiations with the EU will remain a high priority on the agenda of MASA. Our ultimate goal is the Republic of Macedonia to become a democratic, economically prosperous and multicultural European country.

- The increasing incorporation of the international dimension in the scientific and artistic work of MASA, through the cooperation with foreign academies, scientific societies and other scientific institutions, through application and work on scientific projects financed by the European funds and the funds of other international financial institutions, also remains our important priority.

Let us congratulate ourselves on the great jubilee – 50 years of the Macedonian Academy of Sciences and Arts.

**БИБЛИОГРАФИЈА НА АКАДЕМИК ЉУПЧО КОЦАРЕВ
(ПО ПОВОД НЕГОВИОТ 60. РОДЕНДЕН)**

ЛЕОНИД ГРЧЕВ

Во периодот од 1986 година до 2015 година – период од 30 години – академик Љупчо Коцарев публикувал 134 публикации во списанија со фактор на влијание, повеќе од 10 публикации во (меѓународни и домашни) списанија без фактор на влијание, и повеќе од 150 публикации во зборници на рецензирани трудови од (меѓународни и домашни) конференции. Тој бил уредник на 6 книги и автор на 6 патенти (одобрени во САД и/или Европа). Неговите публикации се цитирани според Web of Science (Science Citation Index) 6.384 пати (6.133 **без себецитирање**), додека според Google Scholar тие се цитирани 11.248 пати (податоци од 25 октомври 2015 година). Според статистиката што ја води издавачот Elsevier, статиите публикувани во Elsevier се видени/пристапени преку порталот на издавачот 18.946 пати (податоци до септември 2015 година). Според статистиката што ја води издавачот Springer, вкупниот број преземени (download) текстови од книгите што ги уредувал акад. Коцарев е 36.500 (податоци заклучно со 2014 година). Табела 1 и Табела 2 ја прикажуваат распределбата на публикациите во списанија со фактор на влијание по години и по списанија/издавачи, соодветно. Академик Коцарев соработувал со истражувачи од многу земји, вклучувајќи ги САД, Русија, Кина, Јапонија, Германија, Италија, Шпанија, Полска, Унгарија, Велика Британија, Австралија, Данска, Холандија, Франција, Канада, Мексико и многу други.

Книги

1. V. In, L. Kocarev et al (Editors), Proceeding of 7th Experimental Chaos Conference, American Institute of Physics, 2004
2. L. Kocarev and G. Vattay (Editors), Complex Dynamics in Communication Networks, Series: Understanding Complex Systems, Springer, 2005
3. L. Kocarev, Z. Galias, and S. Lian (Editors), Intelligent Computing Based on Chaos, Series: Studies in Computational Intelligence, Springer, 2009
4. L. Kocarev and S. Lian (Editors), Chaos-based Cryptography: Theory, Algorithms and Applications, Series: Studies in Computational Intelligence, Springer, 2011

5. L. Kocarev (Editor) Proceedings of ICT Innovations 2011, Series: Advances in Intelligent and Soft Computing, Springer, 2012

6. L. Kocarev (Editor) Consensus and Synchronization in Complex Networks, Series: Understanding Complex Systems, Springer, 2013

Публикации во меѓународни списанија со фактор на влијание:

1986

- V. Urumov and L. Kocarev, “The Phase Diagram of a Piecewise-Linear Oscillator”, ZAMM, Z. Angew. Math. Mech., Vol. 66, pp. 396 – 398, 1986

1987

- L. Kocarev, “The Basin Boundaries of One-Dimensional Maps”, Physics Letters A, Vol. 121, pp. 274 – 278,

1987

- L. Kocarev, “Quasifractal Metamorphosis of 1-D Maps,” Physics Letters A, Vol. 125, pp. 389 – 342, 1987

1988

- L. Kocarev, “On a Class of Symmetrical Chaotic Attractors,” Physics Letters A, Vol. 130, pp. 7 – 10, 1988

1990

- V. Urumov and L. Kocarev, “New Metric Elements of the Universal Fine Structure due to Multifurcation,” Physics Letters A, Vol. 144, pp. 220 – 223, 1990

1992

- L. O. Chua, L. Kocarev, K. Eckart, and M. Itoh, “Experimental synchronization in Chua’s circuit”, International Journal of Bifurcation and Chaos, Vol. 2, pp. 705 – 708, 1992
- L. Kocarev, K. S. Halle, K. Eckert, L. O. Chua, and U. Parlitz,

“Experimental demonstration of secure communication via chaotic synchronization”, *International Journal of Bifurcation and Chaos*, Vol. 2, pp. 709 – 713, 1992

- U. Parlitz, L. O. Chua, L. Kocarev, K. S. Halle, and A. Shang, “Transmission of digital signals by chaotic synchronization”, *International Journal of Bifurcation and Chaos*, Vol. 2, pp. 973 – 977, 1992

1993

- L. Kocarev and L. O. Chua, “On chaos in digital filters: case $b = -1$ ”, *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, Vol. 40, pp. 404 – 407, 1993
- L. Kocarev, L. Karadzinov and L. O. Chua, “N-dimensional canonical Chua’s circuit”, *Journal of Circuits, Systems and Computers*, Vol. 3, pp. 239 – 258, 1993
- L. O. Chua, M. Itoh, L. Kocarev and K. Eckart, “Chaos synchronization in Chua’s circuit”, *Journal of Circuits, Systems and Computers*, Vol. 3, pp. 93 – 108, 1993
- V. Belykh, N. N. Verichev, L. Kocarev and L. O. Chua, “On chaotic synchronization in a linear array of Chua’s circuits”, *Journal of Circuits, Systems and Computers*, Vol. 3, pp. 579 – 589, 1993
- T. Kapitaniak, L. Kocarev and L. O. Chua, “Controlling chaos without feedback and control signals”, *International Journal of Bifurcation and Chaos*, Vol. 3, pp. 459 – 568, 1993
- L. Kocarev, A. Shang and L. O. Chua, “Transitions in dynamical regimes by driving: a unified method of control and synchronization of chaos”, *International Journal of Bifurcation and Chaos*, Vol. 3, pp. 479 – 483, 1993
- L. Kocarev, K. S. Halle, K. Eckart and L. O. Chua, “Experimental observation of anti-monotonicity in Chua’s circuit”, *International Journal of Bifurcation and Chaos*, Vol. 3, pp. 1051 – 1055, 1993

1994

- L. Kocarev, D. Dimovski, Z. Tasev and L. O. Chua, “Knotted periodic orbits in Chua’s circuit”, *International Journal of Bifurca*

tion and Chaos, Vol. 4, pp. 609 – 621, 1994

- L. Kocarev and Z. Tasev, “Analytical Description of a Fractal Set Generated by the time-delayed Chua’s Circuit”, International Journal of Bifurcation and Chaos, Vol. 4, pp. 1639 – 1643, 1994
- L. Kocarev, Z. Tasev, and D. Dimovski, “Topological Description of Chaotic Attractors with Spiral Structure”, Physics Letters A, Vol. 190, pp. 399 – 402, 1994
- L. Kocarev, “Chaotic Behavior in Digital Filters”, Journal of the Franklin Institute, Vol. 331B, pp. 937 – 955, 1994

1995

- L. Kocarev and T. Kapitaniak, “On an equivalence of chaotic attractors”, J. Phys. A: Math. Gen., Vol. 28, pp. L249 – L254, 1995
- L. Kocarev and U. Parlitz, “General Approach for Chaotic Synchronization with Application to Communication”, Physical Review Letters, Vol. 74, pp. 5028 – 5031, 1995
- L. Kocarev and T. Stojanovski, “On Chaotic Synchronization and Secure Communication”, IEICE Transactions on Fundamentals and Electronics, Communications and Computer Science, Vol. J78-A, pp. 1142 – 1147, 1995
- L. Kocarev and P. Janjic, “On Turing Instability in a System of Diffusely Coupled Oscillators”, IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, Vol. 42, pp. 779 – 784,

1995

- J. Brindely, T. Kapitaniak, and L. Kocarev, “Controlling Chaos by Chaos in Geophysical Systems”, Geophysical Research Letters, Vol. 22, pp. 1257 – 1260, 1995

1996

- L. Kocarev and U. Parlitz, “Generalized Synchronization, Predictability and Equivalence of Unidirectionally Coupled Systems”, Physical Review Letters, Vol. 76, pp. 1816 – 1819, 1996

-
- L. Kocarev, C. W. Wu, and L. O. Chua, “Complex behavior in digital filters: analytical results”, *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, Vol. 43, pp. 234 – 246, 1996
 - U. Parlitz and L. Kocarev, “Multichannel Communication Using Auto - Synchronization”, *International Journal of Bifurcation and Chaos*, Vol. 6, pp. 581 – 591, 1996
 - L. Kocarev and T. Stojanovski, “Linear Conjugacy of Vector Fields in Lur’e Form”, *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, Vol. 43, pp. 782 – 785, 1996
 - U. Parlitz, L. Kocarev, T. Stojanovski and H. Preckel, “Encoding Messages Using Chaotic Synchronization”, *Physical Review E*, Vol. 53, pp. 4351 – 4361, 1996
 - L. Kocarev, U. Parlitz and T. Stojanovski, “An Application of Synchronized Dynamic Arrays”, *Physics Letters A*, Vol. 217, pp. 280 – 284, 1996
 - U. Parlitz, L. Junge, W. Lauterborn and L. Kocarev, “Experimental observation of phase synchronization”, *Physical Review E*, Vol. 54, pp. 2115 – 2117, 1996
 - T. Stojanovski, L. Kocarev and U. Parlitz, “Driving and synchronizing by chaotic impulses”, *Physical Review E*, Vol. 54, pp. 2128 – 2131, 1996
 - L. Kocarev and U. Parlitz, “Synchronizing spatiotemporal chaos in coupled nonlinear oscillators”, *Physical Review Letters*, Vol. 77, pp. 2206 – 2209, 1996
 - U. Parlitz, L. Junge and L. Kocarev, “Synchronization based parameter estimation from time series”, *Physical Review E*, Vol. 54, pp. 6253 – 6259, 1996
 - T. Stojanovski, L. Kocarev and U. Parlitz, “A simple method to reveal the parameters of the Lorenz system”, *International Journal of Bifurcation and Chaos*, Vol. 6, pp. 2645 – 2652, 1996
 - D. Gligorovski, D. Dimovski, L. Kocarev, V. Urumov and L. O.

Chua, “A method for encoding messages by time targeting of the trajectories of chaotic systems”, *International Journal of Bifurcation and Chaos*, Vol. 6, pp. 2119 – 2125, 1996

1997

- T. Stojanovski, L. Kocarev, U. Parlitz, and R. Harris, “Sporadic driving of dynamical systems”, *Physical Review E*, Vol. 55, pp. 4035 – 4048, 1997
- T. Stojanovski, L. Kocarev, and U. Parlitz, “Digital Coding via Chaotic Systems”, *IEEE Trans. on Circuits and Systems, Part I*, Vol. 44, pp. 562 – 565, 1997
- L. Kocarev, Z. Tasev and U. Parlitz, “Synchronizing spatiotemporal chaos of partial differential equations”, *Physical Review Letters*, Vol. 79, pp. 51 – 54, 1997
- L. Kocarev, U. Parlitz, T. Stojanovski, and P. Janjic, “Controlling Spatiotemporal Chaos in Coupled Nonlinear Oscillators”, *Physical Review E.*, Vol. 56, pp. 1238 – 1241, 1997
- T. Stojanovski, U. Parlitz, L. Kocarev, and R. Harris, “Exploiting Delay Reconstruction for Chaos Synchronization”, *Physics Letters A*, Vol. 233, pp. 139 – 152, 1997
- T. Stojanovski, L. Kocarev, and R. Harris, “Applications of Symbolic Dynamics in Chaos Synchronization”, *IEEE Trans. on Circuits and Systems, Part I*, Vol. 44, pp. 1014 – 1018, 1997
- U. Parlitz, L. Kocarev, T. Stojanovski, and L. Junge, “Chaos Synchronization Using Sporadic Driving”, *Physica D*, Vol. 109, pp. 139 – 152, 1997
- L. Kocarev, Z. Tasev, T. Stojanovski and U. Parlitz, “Synchronizing spatiotemporal chaos”, *CHAOS*, Vol. 7, pp. 635 – 643, 1997
- U. Parlitz and L. Kocarev, “Using surrogate data analysis for unmasking chaotic communication systems”, *International Journal of Bifurcation and Chaos*, Vol. 7, pp. 407 – 413, 1997
- U. Parlitz, L. Junge, and L. Kocarev, “Subharmonic entrainment of unstable period orbits and generalized synchronization”, *Physical Review Letters*, Vol. 79, pp. 3158 – 3161, 1997.

1998

- L. Kocarev, U. Parlitz, and B. Hu, “Lie derivatives and dynamical systems”, *Chaos, Solitons and Fractals*, Vol. 9, pp. 1445 – 1455, 1998
- L. Kocarev, P. Janjic, U. Parlitz, and T. Stojanovski, “Controlling spatiotemporal chaos in coupled oscillators by sporadic driving”, *Chaos, Solitons, and Fractals*, Vol. 9, pp. 283 – 293, 1998
- C. Mitrevski, L. Kocarev and N. Jonoska, “On a class of N - dimensional digital filters operating out of its stability region”, *International Journal of Circuit Theory and Application*, Vol. 26, pp. 199 – 205, 1998

1999

- U. Parlitz, L. Junge and L. Kocarev, “Nonidentical synchronization of identical systems”, *International Journal of Bifurcation and Chaos*, Vol. 9, pp. 2305 – 2309, 1999
- L. Junge, U. Parlitz, Z. Tasev and L. Kocarev, “Synchronization and control of spatially extended systems using sensor coupling”, *International Journal of Bifurcation and Chaos*, Vol. 9, pp. 2265 – 2270, 1999

2000

- L. Kocarev, U. Parlitz, and R. Brown, “Robust chaos synchronization”, *Physical Review E*, Vol. 61, pp. 3716 – 3720, 2000
- R. Brown and L. Kocarev, “A unifying definition of synchronization for dynamical systems”, *CHAOS*, Vol. 10, pp. 344 – 349, 2000
- Z. Tasev, L. Junge, U. Parlitz, and L. Kocarev “Synchronization of Kuramoto-Sivashinsky equations using local coupling”, *International Journal of Bifurcation and Chaos*, Vol. 10(4), pp. 869 – 874, 2000
- L. Kocarev and D. Walker, “Compactness of symbolic sequences from chaotic systems”, *Phys. Letters A*, Vol. 274, pp. 200 – 205, 2000

2001

- G. Jakimovski and L. Kocarev, “Chaos and Cryptography: Block Encryption Ciphers Based on Chaotic Maps” IEEE Trans. on Circuits and Systems, Part I, Vol. 48(2), pp. 163 – 169, 2001
- T. Stojanovski and L. Kocarev, “Chaos Based Random Number Generators Part I: Analysis”, IEEE Trans. on Circuits and Systems, Part I, Vol. 48(3), pp. 281 – 288, 2001
- T. Stojanovski, J. Pihl and L. Kocarev, “Chaos Based Random Number Generators Part II: Practical Realization”, IEEE Trans. on Circuits and Systems, Part I, Vol. 48(3), pp. 382 – 385, 2001
- C. Mitrovski and L. Kocarev, “Periodic Trajectories in Piecewise-Linear Maps, IEEE Trans. on Circuits and Systems, Part I, Vol. 48(10), pp. 1244 – 1246, 2001
- L. Kocarev and G. Jakimovski, “Logistic map as a block encryption algorithm”, Phys. Letters A, Vol. 289, pp. 199 – 206, 2001
- L. Kocarev, “Chaos-Based Cryptography: a Brief Overview”, IEEE Circuits and Systems Magazine, Vol. 1(3), pp. 6 – 21, 2001
- G. Jakimoski and L. Kocarev, “Analysis of Some Recently Proposed Chaos-Based Encryption Algorithms,” Phys. Letters A, Vol. 291, pp. 381 – 384, 2001

2002

- L. Kocarev and Z. Tasev, “Lyapunov Exponents, Noise-induced Synchronization and Parrondo’s Paradox,” Physical Review E, 65, pp. 046215 (4 pages), 2002
- Z. Tasev and L. Kocarev, “Performance Evaluation of CPPM Modulation in Multi-path Environments,” Chaos, Fractals and Solitons, Vol. 15, pp. 319 – 326, 2002
- L. Illing, J. Brocker, L. Kocarev, U. Parlitz, and H. D. I. Abarbanel, “When are synchronization errors small?” Physical Review E, 66, pp. 036229, 2002
- L. Kocarev, Z. Tasev, and A. Vardy, “Improving turbo codes by control of transient chaos in turbo-decoding algorithms,” Electron

ics Letters, Vol. 38(20), pp. 1184 – 1186, 2002

2003

- G. Jakimoski and L. Kocarev, “Differential and Linear Probabilities of a Block Encryption Cipher,” IEEE Trans. on Circuits and Systems, Part I, Vol. 50(1), pp. 121 – 123, 2003
- L. Kocarev and G. Jakimoski, “Unpredictable Pseudo-Random Bits Generated by Chaotic Maps,” IEEE Trans. on Circuits and Systems, Part I, Vol. 50(1), pp. 123 – 126, 2003

2004

- P. Popovski, L. Kocarev, and A. Risteski, “Design of Flexible-Length S-Random Interleaver for Turbo Codes,” IEEE Communications Letters, Vol. 8, No. 7, pp. 461 – 463, 2004
- L. Kocarev, M. Sterjev, A. Fekete and G. Vattay, “Public-key Encryption with Chaos,” CHAOS, Vol. 14 (4), pp. 1078 – 1082, 2004.
- L. Kocarev and Z. Tasev, “Chaos and control of transient chaos in turbo-decoding algorithms,” International Journal of Bifurcation and Chaos, Vol. 14, No. 3, pp. 1147 – 1153, 2004
- L. Kocarev and J. Szczepanski, “Finite-space Lyapunov exponents and pseudo-chaos,” Phys Rev Letters, Vol. 93, pp. 234101(1 – 4), 2004

2005

- J. Szczepanski, J.M. Amigo, T. Michalek, and L. Kocarev, “Cryptographically secure substitutions based on the approximation of mixing maps,” IEEE Transactions on Circuits and Systems part I, Vol. 52, No. 2, pp. 443 – 453, 2005
- P. Bergamo, P. D’Arco, A. De Santis and L. Kocarev, “Security of Public Key Cryptosystems based on Chebyshev Polynomials,” IEEE Transactions on Circuits and Systems part I, Vol. 52, No. 7, pp. 1382 – 1393, 2005
- L. Kocarev and P. Amato, “Synchronization in Power-Law Networks,” CHAOS, Vol. 15, pp. 024101(1 – 6), 2005

- L. Kocarev, J. Makraduli, and P. Amato, “Public-Key Encryption Based on Chebyshev Polynomials,” *Circuits, Systems and Signal Processing*, Vol. 24, No. 5, pp. 497 – 517, 2005
- R. Femat, L. Kocarev, L. van Gerven, and M. E. Monsivais-Perez, “Towards Generalized Synchronization of Strictly Different Chaotic Systems,” *Physics Letters A*, Vol. 342, pp. 247 – 255, 2005
- J.M. Amigo, J. Szczepanski, and L. Kocarev, “Cryptographically Secure Substitutions Based on Chaotic Maps,” *Physics Letters A*, Vol. 343, pp. 55 – 60, 2005
- J.M. Amigo, M.B. Kennel and L. Kocarev, “The permutation entropy rate equals the Kolmogorov-Sinai entropy rate for mixing dynamical systems,” *Physica D*, Vol. 210, pp. 77 – 95, 2005

2006

- J.M. Amigo, L. Kocarev, and J. Szczepanski, “Order patterns”, *Physics Letters A*, Vol. 355, pp. 27 – 31, 2006
- A. Fekete, G. Vattay, and L. Kocarev, “Distribution of edge load in scale-free trees” *Physical Review E*, Vol. 73, 046102 (11 pages), 2006
- L. Kocarev, F. Lehmann, G.M. Maggio, B. Scanavino, Z. Tasev, and A. Vardy, “Nonlinear Dynamics of Iterative Decoding Systems: Analysis and Applications,” *IEEE Transactions on Information Theory*, Vol. 52(4), pp. 1366 – 1384, 2006
- N. Masuda, G. Jakimoski, K. Aihara, and L. Kocarev, “Chaotic Block Ciphers: from theory to practical algorithms,” *IEEE Transactions on Circuits and Systems*, Vol. 53(6), pp. 1341 – 1352, 2006
- L. Kocarev, J. Szczepanski, J. M. Amigo, and I. Tomovski, “Discrete Chaos – I: Theory”, *IEEE Transactions on Circuits and Systems*, Vol. 53(6), pp. 1300 – 1309, 2006
- D. Yu, M. Righero, and L. Kocarev, “Estimating topology of networks,” *Physical Review Letters*, Vol. 97, pp. 188701 (4 pages), 2006 (also published in *Virtual Journal of Biological Physics Research*).

2007

- J. M. Amigo, L. Kocarev, and I. Tomovski, “Discrete entropy,” *Physica D*, Vol. 228, pp. 77 – 85, 2007
- J. M. Amigo, L. Kocarev, and J. Szczepanski, “Theory and Practice of Chaotic Cryptography,” *Physics Letters A*, Volume 366, Issue 3, pp. 211 – 216, 2007
- J. M. Amigo, L. Kocarev, J. Szczepanski, “Discrete Lyapunov exponent and resistance to differential cryptanalysis,” *IEEE Transactions on Circuits and Systems*, part II, Volume 54, Issue 10, pp. 882 – 886, October 2007
- L. Pejov, M. La Rosa, and L. Kocarev, “Dynamics of the central phenylene ring torsional motion in phenylene ethynylene oligomers. A quantum - theoretical contribution to the study of conformational basis for single – molecule switching phenomena,” *Chemical Physics*, Vol. 340, Issue 1-3, pp. 1 – 11, 2007
- G. S. Duane, D. Yu, and L. Kocarev, “Identical synchronization, with translation invariance, implies parameter estimation,” *Physics Letters A*, Volume 371, Issues 5-6, 26, Pages 416 – 420, 2007

2008

- P. Checco, M. Biey, and L. Kocarev, “Synchronization in random networks with given expected degree sequences,” *Chaos, Solitons, and Fractals*, Vol. 35, pp. 562 – 577, 2008
- J. Amigo, L. Kocarev, and J. Szczepanski, “On some properties of the discrete Lyapunov exponent”, *Physics Letters A*, Volume 372, Issue 41, Pages 6265 – 6268, 2008
- Y. Liu, K.S. Tang and L. Kocarev, “An adaptive observer design for auto-synchronization of Lorenz System,” *International Journal of Bifurcation and Chaos*, Volume 18, No 8, pp. 2415 – 2423, 2008
- I. Petreska, L. Pejov, and L. Kocarev, “Controlling the torsional stochastic switching in phenylene ethynylene oligomer molecules by external electrostatic fields”, *Phys. Rev. B*, Volume 78, 045209 (5 pages), 2008 (also published in *Virtual Journal*)

- P. Checco, M. Righero, M. Biey, and L. Kocarev, “Synchronization in Networks of Hindmarsh–Rose Neurons”, *IEEE Transactions on Circuits and Systems II: Express briefs*, Vol. 55, No. 12, pp. 1274 – 1278, 2008

2009

- T. Addabbo and L. Kocarev, “Periodic Dynamics in State Dependent Queuing Networks”, *Chaos, Solitons, and Fractals*, Volume 41, Issue 4, Pages 2178 – 2192, 2009
- Y. Mao, W. Tang, Y. Liu and L. Kocarev, “Identification of biological neurons using adaptive observers”, *Cognitive processing*, Vol. 10 (Suppl 1), pages: 41 – 53, 2009
- N. Zlatanov and L. Kocarev, “Random walks on networks: Cumulative distribution of cover time”, *Physical Review E* 80, 041102 (8 pages), 2009
- Y. Liu, Y. Mao, W. Tang, L. Kocarev, “Communication schemes by dynamical minimization algorithm” *International Journal of Bifurcation and Chaos*, Volume 19, Issue 7 Pages 2429 – 2437, 2009

2010

- D. Smilkov and L. Kocarev, “Rich-club and page-club coefficients for directed graphs”, *Physica A*, Volume 389, Pages 2290 – 2299, 2010
- I. Petreska, I. Tomovski, E. Gutierrez, L. Kocarev, F. Bono, K. Poljansek, “Application of modal analysis in assessing attack vulnerability of complex networks”, *Communications in Nonlinear Science and Numerical Simulation* (Elsevier), Volume 15, Issue 4, Pages 1008 – 1018, 2010
- D. Trpevski, W. K. S. Tang, and L. Kocarev, “Model for rumor spreading over networks”, *Physical Review E* 81, 056102 (11 pages), 2010
- D. Trpevski, D. Smilkov, I. Mishkovski, and L. Kocarev, “Vulnerability of labeled networks”, *Physica A: Statistical Mechanics and its Applications* Volume 389, Issue 23, Pages 5538 – 5549, 2010

- L. Kocarev, N. Zlatanov and D. Trajanov, “Vulnerability of networks of interacting Markov chains”, *Phil. Trans. R. Soc. A* 368, 2205 – 2219, 2010
- L. Kocarev and V. In, “Network science: A new paradigm shift”, *IEEE Network*, vol. 24(6), pages: 6 – 9, 2010

2011

- M. Mishkovski, M. Biey, and L. Kocarev, “Vulnerability of complex networks”, *Communications in Nonlinear Science and Numerical Simulation* (Elsevier), Volume 16, Issue 1, Pages 341 – 349, 2011
- I. Petreska, L. Pejov, and L. Kocarev, “Exploring the possibilities to control the molecular switching properties and dynamics: A field-switchable rotor-stator molecular system”, *Journal of Chemical Physics* 134, 014708 (12 pages), 2011
- D. Smilkov and L. Kocarev, “Analytically solvable processes on networks”, *Physical Review E* 84, 016104 (8 pages) 2011
- T. Addabbo, A. Fort, L. Kocarev, S. Rocchi, V. Vignoli, “Pseudo-Chaotic Lossy Compressors for True Random Number Generation”, *IEEE Transactions on Circuits and Systems I: Regular Papers*, Volume 58, Issue 8, Page(s): 1897 – 1909, 2011
- A. Stanoev, D. Smilkov, L. Kocarev, “Identifying communities by influence dynamics in social networks”, *Physical Review E*, 84, 046102 (10 pages), 2011
- M. Miskovski, M. Righero, M. Biey, and L. Kocarev, “Enhancing robustness and synchronizability of networks homogenizing their degree distribution”, *Physica A*, Vol. 390, Issues 23-24, p. 4610 – 4620, 2011

2012

- D. Smilkov and L. Kocarev, “Influence of the network topology on epidemic spreading”, *Physical Review E* 85, 016114 (10 pages), 2012
- I. Tomovski and L. Kocarev, “Simple Algorithm for Virus Spreading Control on Complex Networks”, *IEEE Transactions on Cir*

uits and Systems I: Regular Papers, Volume: 59, Issue: 4, Page(s): 763 – 771, 2012

- M. Mirchev, G. Duane, W. Tang, and L. Kocarev, “Improved modeling by coupling imperfect models”, *Communications in Nonlinear Science and Numerical Simulation* (Elsevier), Volume 17, Pages 2741 – 2751, 2012
- L. Basnarkov and L. Kocarev, “Forecast improvement in Lorenz 96 system”, *Nonlinear Processes in Geophysics*, Volume 19, 569–575, 2012

2013

- I Trpevski, L Basnarkov, D Smilkov, L Kocarev, “Empirical correction techniques: analysis and applications to chaotically driven low-order atmospheric models”, *Nonlinear Processes in Geophysics*, Volume 20 Issue 2 Pages 199-206, 2013
- A Kanevce, I Mishkovski, L Kocarev, “Modeling long-term dynamical evolution of Southeast European power transmission system”, *Energy*, Volume 57, Pages 116-124, 2013
- T. Stojanovski, L. Kocarev, “Construction of Markov Partitions in PL1D Maps”, *IEEE Transactions on Circuits and Systems II: Express Briefs*, Volume 60, Issue 10, Page(s): 702 – 706, 2013

2014

- L Basnarkov, GS Duane, L Kocarev, “Generalized synchronization and coherent structures in spatially extended systems”, *Chaos, Solitons & Fractals* 59, 35 – 41, 2014
- I. Tomovski, I. Trpevski, L. Kocarev, “Topology independent SIS process: An engineering viewpoint”, *Communications in Nonlinear Science and Numerical Simulation*, Vol. 19, pp. 627 – 637, 2014
- A. Bogojeska, M. Mirchev, I. Mishkovski, L. Kocarev, “Synchronization and Consensus in State-Dependent Networks”, *IEEE Transactions on Circuits and Systems I: Regular Papers*, Volume: 61, Issue: 2, Page(s): 522 – 529, 2014
- M. Mirchev, L. Basnarkov, F. Corinto, L. Kocarev, “Cooperative

Phenomena in Networks of Oscillators with Non-identical Interaction and Dynamics”, IEEE Transactions on Circuits and Systems I: Regular Papers, Volume: 61, Issue: 3, Page(s): 811 – 819, 2014

- D. Smilkov, C. A. Hidalgo, and L. Kocarev, “Beyond network structure: How heterogeneous susceptibility modulates the spread of epidemics”, Scientific Reports 4, Article number: 4795, Published 25 April 2014, doi:10.1038/srep04795, 2014
- A. Tenev, S. Markovska-Simoska, L. Kocarev, J. Pop-Jordanov, A. Müller, and G. Candria, “Machine learning approach for classification of ADHD adults”, International Journal of Psychophysiology, (available online 26 January 2013), Vol. 93, pp. 162 – 166, 2014
- A. Stanoev, D. Trpevski, and L. Kocarev, “Modeling the Spread of Multiple Concurrent Contagions on Networks”, PLoS ONE 9(6): e95669. doi:10.1371/journal.pone.0095669, Published: June 12, 2014
- K. Trivodaliev, A. Bogojeska, and L. Kocarev, “Exploring Function Prediction in Protein Interaction Networks via Clustering Methods”, PLoS ONE 9(6): e99755. doi:10.1371/journal.pone.0099755, Published: June 27, 2014
- A. Gajduk, M. Todorovski, and L. Kocarev, “Stability of power grids: An overview”, Eur. Phys. J. Special Topics, Volume 223, Issue 12, pp 2387-2409, October 2014,
- A. Gajduk, M. Todorovski, J. Kurths and L. Kocarev, “Improving power grid transient stability by plug-in electric vehicles”, New J. Phys. 16 115011, Published 11 November 2014
- I. Trpevski, A Stanoev, A Koseska and L Kocarev, “Discrete-time distributed consensus on multiplex networks”, New J. Phys. 16, 113063, Published 26 November 2014

2015

- V. Zdraveski, M. Todorovski, and L. Kocarev, “Dynamic intelligent load balancing in power distribution networks”, *International Journal of Electrical Power & Energy Systems*, Volume 73, Pages 157–162, December 2015
- I. Tomovski and L. Kocarev, “Network topology inference from infection statistics”, *Physica A: Statistical Mechanics and its Applications*, Volume 436, Pages 272–285, 15 October 2015
- M. Jovanovik, A. Bogojeska, D. Trajanov, and L. Kocarev, “Inferring Cuisine - Drug Interactions Using the Linked Data Approach”, *Scientific Reports* 5, Article number: 9346, 2015
- K. Trivodaliev, S. Kalajdziski, I. Ivanoska, B. Risteska Stojkoska, and L. Kocarev, “SHOPIN: Semantic Homogeneity Optimization in Protein Interaction Networks”, *Advances in Protein Chemistry and Structural Biology*, Available online 24 August 2015
- Y. Tang, X. Xing, H. Karimi, L. Kocarev, and J. Kurths, “Tracking Control of Networked Multi-Agent Systems under New Characterizations of Impulses and its Applications in Robotic Systems”, *IEEE Transactions on Industrial Electronics*, Date of Publication: 08 July 2015

Распределба по години

1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	Total
1	2	1	0	1	0	3	7	4	5	24

1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	Total
12	10	3	2	4	7	4	2	4	7	55

2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	Total
6	5	5	4	6	6	4	3	11	5	55

Списание (издавач)	Број на публикации
Advances in Protein Chemistry and Structural Biology (Springer)	1
Chaos: An International Journal of Nonlinear Science (American Institute of Physics)	4
Chaos, Solitons, and Fractals (Elsevier)	6
Chemical Physics (Elsevier)	1
Circuits; Systems and Signal Processing (Springer)	1
Cognitive processing (Springer)	1
Communications in Nonlinear Science and Numerical Simulation (Elsevier)	4
Electronics Letters (The Institution of Engineering and Technology, England)	1
Energy (Elsevier)	1
Eur. Phys. J. Special Topics (Springer)	1
Geophysical Research Letters (Wiley)	1
IEEE Circuits and Systems Magazine (IEEE, USA)	1
IEEE Communication Letters (IEEE, USA)	1
IEEE Network (IEEE, USA)	1
IEEE Transaction on Circuits and Systems (IEEE, USA)	23
IEEE Transactions on Industrial Electronics (IEEE, USA)	1
IEEE Transaction on Information Theory (IEEE, USA)	1
IEICE Transactions on Fundamentals and Electronics, Communications and Computer Science (The Institute of Electronics, Information and Communication Engineers, Japan)	1
International Journal of Bifurcation and Chaos (World Scientific)	18
International Journal of Circuit Theory and Application (Wiley)	1
International Journal of Electrical Power & Energy Systems (Elsevier)	1
International Journal of Psychophysiology (Elsevier)	1
Journal of Chemical Physics (American Institute of Physics)	1
Journal of Circuits, Systems, and Computers (World Scientific)	3
Journal of the Franklin Institute (Elsevier)	1
Journal of Physics A: Mathematical and Theoretical (Institute of Physics, UK)	1
New Journal of Physics (Institute of Physics, UK)	2
Nonlinear Processes in Geophysics (European Geosciences Union)	2
Physica A: Statistical Mechanics and its Applications (Elsevier)	4
Physica D: Nonlinear phenomena (Elsevier)	3
Physical Review E (American Physical Society)	15
Physical Review B (American Physical Society)	1
Physical Review Letters (American Physical Society)	7
Physics Letters A (Elsevier)	16
PLoS ONE (Public Library of Science)	2
Scientific Reports (Nature)	2
Z. Angew. Math. Mech (Wiley)	1
Вкупно	134

SPECIAL CASES OF NETWORK INFERENCE FROM INFECTION STATISTICS

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Abstract: Following our recent work on inferring network topology from infection statistics [1], in this paper we analyze two special cases to which the methodology presented in [1] can be applied with improved efficiency. In the first case we suggest and analyze the use of discriminators when it is known that the observed network is unweighted. Next, we analyze the case where the physical topology of a weighted network is known (there is a prior knowledge about how nodes are connected) and we need to determine link weights. In both cases, significant improvement in times required for proper assessment of the network topology is obtained, compared to the general case. The results suggest that the methodology presented in [1] is of practical use in cases where we have some prior knowledge about the network structure.

1. Introduction

The topic of inference of the elements of networked structures, as a sub-domain of a broader set of problems, dealing with discovering underlying mechanisms that drive complex systems, has been of particular interest in the last few years. This, followed the decade in which studies of complex networks were considered as of at-most importance. The interest in complex networks, and phenomena related to them, though not new, was raised primarily by the emergence of the Internet and its by-products, mostly the social networks and network applications. Scientists were suddenly able to observe in real-time how new and different forms of networks (technical and social)

were forming, followed by spontaneous development of dynamical processes taking place on them. Parallels between natural forms and technical networks were drawn and the research of vast variety of natural and social networked structures and phenomena blossomed. Major achievements were made in the study of brain networks [2, 3, 4, 5], epidemic spreading processes [6, 7, 8, 9], gene and protein networks. In parallel, methods and models giving more detailed insight were developed, as well as control algorithms suggested dealing with spontaneous and emerging phenomena in technical structures, like cascading failures and blackouts of the infrastructure grids [10, 11, 12, 13, 14].

In that context, research related to inference of network topology, as an inverse problem, was a natural extension to the on-going work in the field of complex networks. Recently, number of works addressed the issue. Most of them were focused on the inference of gene-regulatory networks [15, 16, 17, 18, 19], where network inference is a natural step that precedes the analysis. Similar statement is valid for the brain networks as well [20, 21, 22]. Finally, number of works dealt with inference of network topology from more theoretical setups [23, 24, 25, 26, 27, 28, 29].

In this article, following our recent work [1], we study inference of network links from infectious statistic for some special cases. In [1], we suggested methodology and developed an algorithm for inference of network topology (link weights) from infectious statistics, providing that the number of nodes and the characteristics of the observed infectious process are known. Though the obtained result is exact, in practice, the time necessary for proper assessment of link weights by the suggested algorithm is excessive. The reason for this lies in the huge statistical sample necessary for convergence of the statistical means. In the conclusion of [1], we mentioned that the method and the implementation algorithm may be adjusted to give results in significantly shorter times, in networks for which we have some prior knowledge. In what follows, We present the results of that study.

2. Previous work

In our previous work [1], We have considered the following problem: given a network for which we know only the number of nodes and their infection status at each instance, to determine the network topology. The observed network is supposed to be weighted and, in general case, unsymmetrical.

The results were obtained for a SIS type of infection with known β

and δ parameters. We used the standard definition for the SIS process, as described in [30, 31, 32, 33, 34, 35, 36, 37]. Due to specifics in the problem definition, we described the SIS infection with a status difference equation:

$$s_i(t+1) = (1 - s_i(t)) \left(1 - \prod_{\substack{j=0, N-1 \\ j \neq i}} (1 - a_{ij}(t)\beta_{ij}(t)s_j(t)) \right) + s_i(t)(1 - \delta_i(t)) \quad (1)$$

In (1) $s_i(t)$ represents the status of node i at the instance t , with $s_i(t) = 0$ denoting that the node is susceptible and $s_i(t) = 1$ that the node is infected.

Network links are represented with the random $\{0, 1\}$ functions $a_{ij}(t)$. The value $a_{ij}(t) = 1$ denotes that node i is connected to node j at time t and may be influenced (among other be infected) by it. It is important to emphasize that the influence denoted by $a_{ij}(t)$ is only in the direction from node j to node i (the reverse influence is denoted by $a_{ji}(t)$). We consider that the rate of contact denoted by these functions follow the normal operating patterns of the network. The occurrence of infection is only a by-product of these normal patterns. The rate of influence is denoted by the expectation $\omega_{ij} = E[a_{ij}(t)]$ and is called weight. In other words, the weights represent the fraction of time in which node j influences node i (is connected to). Networks following this link-behavioral pattern are known as temporal networks. For details regarding the temporal networks we refer the readers to [38, 39, 40]

The terms $\beta_{ij}(t)$ are $\{0, 1\}$ random functions that define whether a susceptible node i when in contact only with an infected node j will become infected, in the time-frame $[t, t + 1]$. We consider (and this is generally accepted approach when studying spreading processes) that expectation values (means in time) of $\beta_{ij}(t)$'s are equal for all $i \neq j$, i.e $\beta_{ij} = E[\beta_{ij}(t)] = \beta$. The parameter β is known as "infection rate". In the same manner, the random functions $\delta_i(t)$ denote whether an infected node i will overcome the infection and become susceptible in the time-frame $[t, t + 1]$. It is custom to call their expectation values $\delta = \delta_{ij} = E[\delta_{ij}(t)]$ curring rate.

Relation (1) is a $\{0, 1\} \rightarrow \{0, 1\}$ mapping that describes the status change of each particular node as function of time and therefore exactly describes the SIS process. Though it may be rarely seen in the literature, it is only an adapted algebraic relation derived from logical expression that defines the SIS process.

Starting from (1), and after a complicated mathematical procedure, We have derived the following relation for the weight of each individual link [1]:

$$w_{ij} = \frac{1}{\beta} \left(1 - \frac{1 - E[X_i^{t+1} | \overline{X}_i^t X_j^t Y_{n,i,j}^t]}{1 - E[X_i^{t+1} | \overline{X}_i^t \overline{X}_j^t Y_{n,i,j}^t]} \right) \quad (2)$$

In (2), \overline{X}_i^t denotes the logical event: node i is susceptible at time t (consequently X_i^t denotes: node i is infected at time t). The term $Y_{n,i,j}^t$ denotes a logical joint event that describes the infection status of all nodes in the network, except nodes i and j . For example: let $i = 0$, $j = 1$, and $N = 5$. Event $Y_{n=5,0,1}^t = X_2^t \overline{X}_3^t X_4^t$ is a joint infection event that describes the status of nodes 2, 3 and 4, stating: at time t node 2 is infected, node 3 is not infected, node 4 is infected. Index n is a variation index, which value ranges between $n = 0$, for the joint event in which all considered nodes are susceptible and $n = 2^{N-2} - 1$ for the joint event where all considered nodes are infected. In the variation count we use lexicographical ordering in which position of each node is determined by its index i in descending order. In our example the lexicographical ordering is $2 \succ 3 \succ 4$, so the variation index is $n = 1_2 \times 2^2 + 0_3 \times 2^1 + 1_4 = 5$.

In the further text we will often disregard the index t , when discussing the infection events as repetitive. We will use the index only when we would like to explicitly connect the events with time t .

For practical implementation, it is sufficient to chose and observe only one of the joint events $Y_{n,i,j}$. In [1] we suggest a method how to approximately determine $Y_{n,i,j}$ (i.e. n) that is expected to have a very high repeatability, and use that event for observation. We call this event "particular event" and denote it with $Y_{p,i,j}$. For details regarding the derivation of equation (2), practical implementation of the obtained result and the terminology used, we refer the readers to [1].

In [1], we argued that though expression (2) is exact, the suggested implementation algorithm is in general impractical, since it requires huge amount of time for proper assessment. This is due to two reasons: large statistical sample necessary to achieve the desired accuracy and the low repeatability of the particular event. As a method to improve on the repeatability of $Y_{p,i,j}$ in [1] we have suggested a modified form of the original implementation algorithm, that we named Virus Overload (VO). This variant understood that the network is overloaded by different types of viruses (or distinguishable

strains of the same virus) with identical viral characteristics β and δ . This is an artificial methodology that may be implemented only on technical networks.

The results of the analysis of the VO methodology suggested significant improvement compared to the original approach, however the required assessment times were still unsatisfactory. In the articles Conclusion we have stated that, repeatability of the particular event (that determines how fast the required statistical sample may be collected) may be further improved in some special cases. In the next section we consider some of these special cases.

3. Problem definition

In this article, We analyze the method and algorithm presented in [1] in two special cases in which we have some a priori knowledge about the network itself. In the first case, we know that the network under observation is unweighted (purely topological network) and we need to find which links exist. In the second we know the underlying topological structure i.e which links exist, but we know nothing about the weights of those links. In what follows, We give a brief description of the application of the methodology presented in [1], with an adequate adaptation, for these two cases.

3.1. Unweighted networks

Since the unweighted networks have link weights that are either $\omega_{ij} = 0$ or $\omega_{ij} = 1$, the method presented in [1] and the expression given with (2) may be adapted with the use of discriminators. Let ω_{ij}^τ be the estimate of the link weight ω_{ij} at time of observation τ obtained using the relation (2). We define $\omega_{ij}^{\tau,d}$ as an estimation of the link weight after the use of the discriminator as:

$$\omega_{ij}^{\tau,d} = \begin{cases} 0, & \text{if } L_0 < \omega_{ij}^\tau < R_0 \\ 1, & \text{if } L_1 < \omega_{ij}^\tau < R_1 \end{cases} \quad (3)$$

In (3) L_1 and R_1 define the lower and the upper limit of ω_{ij}^τ for which the discriminator will decide that the link (i, j) exists. The same stands in regard to L_0 and R_0 and the decision that a link does not exist. Though it is customary to set similar type of discriminators using simple thresholds (usually $L_0 = -\infty$, $R_0 = L_1 = 0.5$, $R_1 = +\infty$), we suggest more complex discriminator form. The reason for this is that in the early stages of assessment

use of simple discriminator may lead to wrong conclusion and a premature termination of the assessment process.

3.2. Known topological structure

In this case we have a major advantage in comparison to the general case. In [1], an assumption is made in the derivation of the equation (2) that all nodes in the network are potentially connected among each other (full mesh). This is a natural hypothesis, since no knowledge exists about which nodes are neighbors of an arbitrary node i , and therefore influence it (may infect it). Then it is expected that the assessment process determines whether this assumption is true or not, by producing values $\omega_{ij} = 0$ if nodes i and j are not connected. In mathematical terms, this resulted in a very complex form of the joint events $Y_{n,i,j}$ (as well as the particular event $Y_{p,i,j}$) that comprised of the statuses of all nodes in the network (except i and j).

If the topological structure is known, there is an a priori knowledge about which nodes may influence (infect) an arbitrary node. Therefore, the initial full mesh hypothesis may be dismissed. In the alternative derivation procedure (omitted due to the identical form of the one presented in [1]) the joint events $Y_{n,i,j}$, comprise only of the statuses of nodes that are neighbors of node i . In practical terms this results in much higher repeatability of the particular event. Furthermore, this repeatability is highly correlated with the node degree d_i . To illustrate this, consider $d_i = 1$; then $Y_{p,i,j}$ is an empty event and may be considered to repeat in every step of the infection dynamics. For $d_i = 2$, $Y_{p,i,j} = X_k$ or $Y_{p,i,j} = \overline{X_k}$ (whatever is more probable), where k is the second neighbor of node i , besides node j ; the repeatability of the $Y_{p,i,j}$ is $\max(E[X_k], E[\overline{X_k}])$ per time step. Since the size of the statistical sample for given time period T is proportional to the repeatability of $Y_{p,i,j}$, the time required for collecting a sufficient amount of data for proper assessment of a link weight is now much shorter. This especially refers to links leading to (in-links) nodes with small node degrees.

As may already be seen, the assessment of each in-link may be performed individually, and without any knowledge about the rest of the network. In that sense, this methodology may be adapted to cases where only segments of a larger network should be inferred.

4. Materials and tools

To evaluate the suggested methodology adaptation, as well as to compare it with the results obtained in [1], in this article we analyze the special cases described in Section 3, on the same networks used in [1]:

- unweighted Barabási–Albert network [41] (UBA, $N = 30$, $m_0 = 3$, $m = 2$);
- unweighted Watts–Strogatz network [42] (UWS, $N = 30$, $r = 3$, $p = 0.2$);
- unweighted Erdős–Rényi network [43] (UER, $N = 30$, WS algorithm [42] with $r = 3$, $p = 0.9$);
- weighted Barabási–Albert network (WBA derived from the UBA network by adding random link weights);
- weighted WattsStrogatz network (WWS derived from the UWS network by adding random link weights);
- weighted Erdős–Rényi network (WER derived from the UER network by adding random link weights);

As stated in [1], the unweighted networks are symmetrical. However, this does not impact the generality of the result in the unweighted case, since each direction (in-link) is treated separately. In the weighted case, the symmetry is lost, due to assignment of different weights for each link direction.

In the same context, as tools for evaluation of the obtained results, we use functions similar to those suggested in [1]. However, due to the specific changes in the circumstances under which the network topology is inferred, we make adequate adaptations.

For the case where we use discriminators, we define two new estimation functions:

- fraction of Correctly Identified Links- $CIL(\tau)$;
- fraction of UnIdentified Links- $UIL(\tau)$;

For the case of known physical topology, we adapt almost identical error functions (EF) and precision estimation functions (PEF) as described in [1], with the necessary correction for the normalization factor, that is now L , rather than $N(N - 1)$, with L being the number of links.

The EF considered are defined as:

- Mean absolute error - $mae(\tau) = \frac{1}{L} \sum_{i=0}^{N-1} \sum_{\substack{j=0 \\ j \neq i}}^{N-1} \|w_{ij} - w_{ij}^\tau\|$;
- Maximal absolute error for an existing link - $xae1(\tau) = \max_{i \neq j, w_{ij} \neq 0} \|w_{ij} - w_{ij}^\tau\|$;

The PEFs are defined with:

$$k_i(\tau) = \frac{\sum_i \sum_{j, j \neq i} \delta(a_i < \|w_{ij} - w_{ij}^\tau\| < b_i)}{L}$$

where $a_i = 10^{-i}$ for $i = 1, 2, 3$, $a_4 = 0$, $b_i = 10^{-i+1}$ for $i = 2, 3, 4$, $b_1 = \infty$.

5. Results

5.1. Discriminator method

In the analysis of the roll of discriminators on the quality of assessment for the unweighed networks, the original time series obtained in [1] were used, except for the case of the UBA graph (in the original work virus overload method was utilized for this graph). Parts of the obtained time assessments were run trough discriminator with following parameters: $L_0 = -0.3$, $R_0 = 0.3$, $L_1 = 0.7$, and $R_1 = 1.3$. The results of the analysis are presented in Fig.1.

The results in all three cases show that almost all links are recognized within a time-frame of approximately 2×10^{10} cycles. Simultaneously, the fraction of unrecognized links approaches zero within the same time frame. This is an improvement compared to the results given for the same networks in [1]. The improvement is expected, since the detection errors in the results presented in [1] were mainly small and equally distributed around the expected means. However, the time required for the assessment is still excessive and , further more, grows with the size of the network.

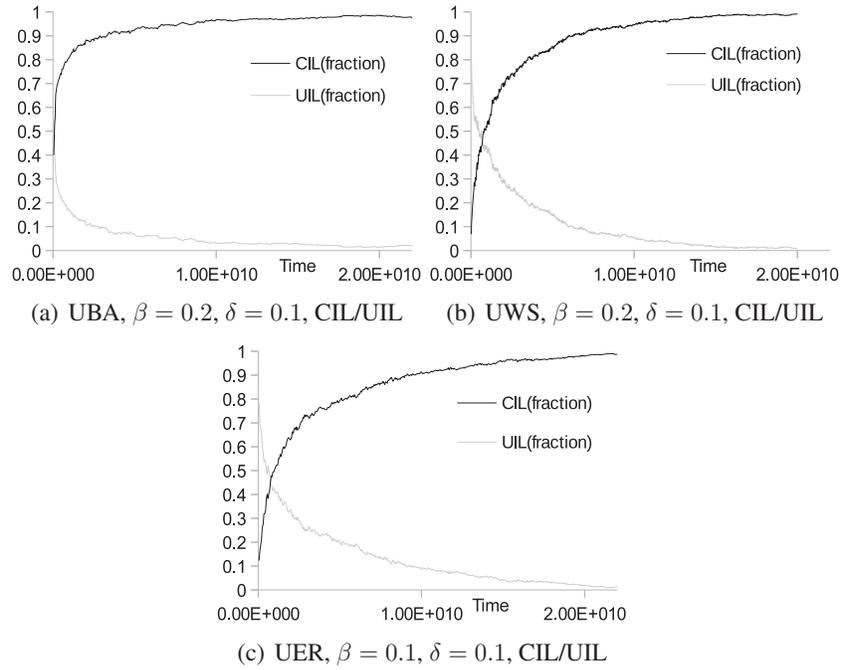


Figure 1: Time evolution of $CIL(\tau)$ and $UIL(\tau)$ for the unweighted networks, when discriminator is used in the assessment process.

5.2. Known physical topology

The results of the analysis of the implementation of the methodology and the algorithm in cases where the physical topology is known, are presented in Figures 2 and 3.

As can be seen from the figures, there is a major improvement in regard to assessment times, compared to the general case [1]. To illustrate this, in Table. 1 we compare times required to reach certain values (marks) of both $mae(t)$ and $xae1(t)$ in the general case and the case where the physical topology is known.

Network	$mae(t)$ time/mark	$xae1(t)$ time/mark
WBA general	$2 \times 10^{11}/0.01$	$2 \times 10^{11}/0.1$
WBA known	$1.3 \times 10^7/0.01$	$1.1 \times 10^7/0.1$
WWS general (VO 200)	$2.2 \times 10^9/0.02$	$10^9/0.1$
WWS known	$7 \times 10^7/0.02$	$8 \times 10^7/0.1$
WER general (VO 200)	$1.4 \times 10^9/0.01$	$1.4 \times 10^9/0.05$
WER known	$3 \times 10^7/0.01$	$6.1 \times 10^7/0.05$

Table 1:

As may be seen from the table, the improvement in the assessment times varies in the 10^3 - 10^4 fold range (the results for WWS and WER in [1] were obtained for VO algorithm which gave a 200 fold improvement in regard to the standard algorithm). Major improvement may be seen for the WBA graph. This comes as no surprise: Barabasi-Albert graph is characterized by a large number of nodes with small node degrees. Using the newly adapted methodology, assessment of the nodes with small node degrees is fast. For those nodes the particular event is uncomplicated: it consists only of few or even no (node degree 1) joint events. Therefore, its repeatability is much higher, compared to the general case [1].

6. Conclusion

In this article, we have analyzed two special cases in which given some prior knowledge about an unknown network, we can utilize assessment methodology described in [1] with improved efficiency. In the first case, we considered networks that are known to be unweighted, and suggested use of discriminators. In the second we have observed the case, where the physical topology

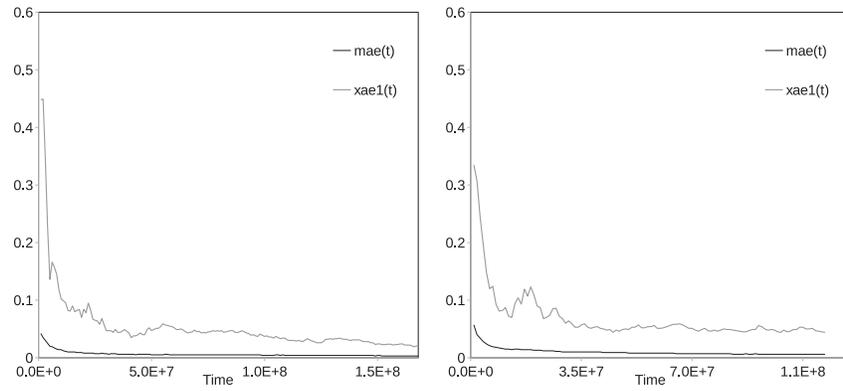
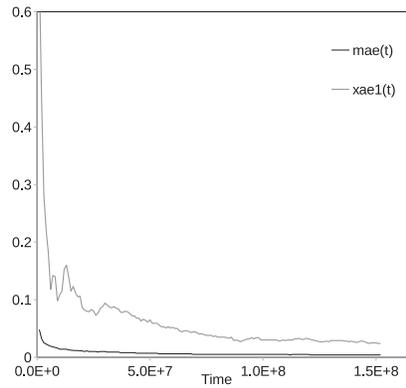
(a) WBA, $\beta = 0.4$, $\delta = 0.1$, EF(b) WWS, $\beta = 0.2$, $\delta = 0.1$, EF(c) WER, $\beta = 0.2$, $\delta = 0.1$, EF

Figure 2: Time evolution of EF obtained from the simulations performed on the unweighted networks in case of a priori known physical topology

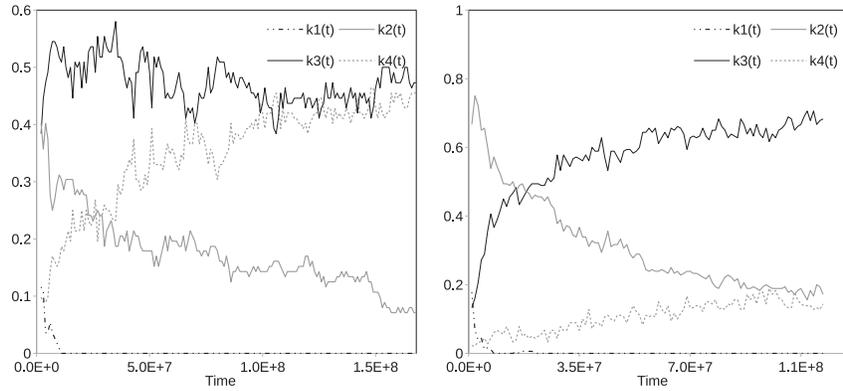
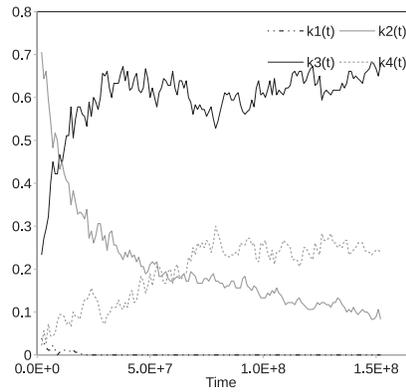
(a) WBA, $\beta = 0.4$, $\delta = 0.1$, PEF(b) WWS, $\beta = 0.2$, $\delta = 0.1$, PEF(c) WER, $\beta = 0.2$, $\delta = 0.1$, PEF

Figure 3: Time evolution of PEF obtained from the simulations performed on the unweighted networks in case of a priori known physical topology

of a network is known. It was shown that simple redefinition of the joint events $Y_{n,i,j}^t$, and therefore the particular event $Y_{p,i,j}^t$, significantly improved the assessment time.

One should consider that in most networks, even in the presented cases, the infectious dynamics is insufficiently fast, compared to real-time processes occurring in those network. Therefore, proper assessment can not be made in, what would considered to be, "normal" time frame, using these methods alone. Combination of the presented algorithms with the VO approach, will lead to further improvement, especially in the case where the physical topology is known. Providing that the maximal node degree of the observed network is low to moderate (below 20), combination of the methodology presented in Section 3.2 and VO may be used for measurement of mid-term network activity, regardless of the size of the network. However, one should bare in mind that, as stated in [1], use of VO would in practice affect the network dynamics as well, so proper correction terms should be considered.

References

- [1] I. Tomovski, L. Kocarev, Network topology inference from infection statistics, *Physica A: Statistical Mechanics and its Applications* 436 (2015) 272 – 285. doi:<http://dx.doi.org/10.1016/j.physa.2015.03.090>.
- [2] O. Sporns, D. R. Chialvo, M. Kaiser, C. C. Hilgetag, Organization, development and function of complex brain networks, *Trends in Cognitive Sciences* 8 (9) (2004) 418 – 425. doi:<http://dx.doi.org/10.1016/j.tics.2004.07.008>.
- [3] E. Bullmore, O. Sporns, Complex brain networks: graph theoretical analysis of structural and functional systems., *Nature reviews. Neuroscience* 10 (3) (2009) 186–198. doi:10.1038/nrn2575.
- [4] V. M. Eguiluz, D. R. Chialvo, G. A. Cecchi, M. Baliki, A. V. Apkarian, Scale-free brain functional networks, *Phys. Rev. Lett.* 94 (1) (2005) 018102.
- [5] D. S. Bassett, E. Bullmore, Small-world brain networks, *The Neuroscientist* 12 (6) (2006) 512–523.

- [6] C. Moore, M. E. J. Newman, Epidemics and percolation in small-world networks, *Phys. Rev. E* 61 (2000) 5678–5682. doi:10.1103/PhysRevE.61.5678.
- [7] M. E. J. Newman, Spread of epidemic disease on networks, *Phys. Rev. E* 66 (2002) 016128. doi:10.1103/PhysRevE.66.016128.
- [8] R. Pastor-Satorras, A. Vespignani, Epidemic spreading in scale-free networks, *Phys. Rev. Lett.* 86 (14) (2001) 3200–3203.
- [9] R. Pastor-Satorras, A. Vespignani, Epidemic dynamics and endemic states in complex networks, *Phys. Rev. E* 63 (6) (2001) 066117.
- [10] H. Ren, I. Dobson, Using transmission line outage data to estimate cascading failure propagation in an electric power system, *Circuits and Systems II: Express Briefs, IEEE Transactions on* 55 (9) (2008) 927–931. doi:10.1109/TCSII.2008.924365.
- [11] B. Carreras, D. Newman, I. Dobson, A. Poole, Evidence for self-organized criticality in a time series of electric power system blackouts, *Circuits and Systems I: Regular Papers, IEEE Transactions on* 51 (9) (2004) 1733–1740. doi:10.1109/TCSI.2004.834513.
- [12] J. Wang, Robustness of complex networks with the local protection strategy against cascading failures, *Safety Science* 53 (2013) 219 – 225. doi:http://dx.doi.org/10.1016/j.ssci.2012.09.011.
- [13] F. Tan, Y. Xia, W. Zhang, X. Jin, Cascading failures of loads in interconnected networks under intentional attack, *EPL (Europhysics Letters)* 102 (2) (2013) 28009.
- [14] I. Hernandez-Fajardo, L. Dueas-Osorio, Probabilistic study of cascading failures in complex interdependent lifeline systems, *Reliability Engineering & System Safety* 111 (2013) 260 – 272. doi:http://dx.doi.org/10.1016/j.ress.2012.10.012.
- [15] T. S. Gardner, D. di Bernardo, D. Lorenz, J. J. Collins, Inferring genetic networks and identifying compound mode of action via expression profiling, *Science* 301 (5629) (2003) 102–105.

- [16] K. Basso, A. A. Margolin, G. Stolovitzky, U. Klein, R. Dalla-Favera, A. Califano, Reverse engineering of regulatory networks in human b cells, *Nature Genetics* 37 (4) (2005) 382–390.
- [17] F. Geier, J. Timmer, C. Fleck, Reconstructing gene-regulatory networks from time series, knock-out data, and prior knowledge, *BMC Systems Biology* 1 (2007) 1–11.
- [18] M. Bansal, V. Belcastro, A. Ambesi-Impiombato, D. di Bernardo, How to infer gene networks from expression profiles, *Molecular Systems Biology* 3 (1). doi:10.1038/msb4100120.
- [19] W. Zhao, E. Serpedin, E. R. Dougherty, Inferring gene regulatory networks from time series data using the minimum description length principle, *Bioinformatics* 22 (17) (2006) 2129–2135. doi:10.1093/bioinformatics/btl364.
- [20] C. Stam, B. van Dijk, Synchronization likelihood: an unbiased measure of generalized synchronization in multivariate data sets, *Physica D: Nonlinear Phenomena* 163 (3-4) (2002) 236–251.
- [21] J. C. Rajapakse, J. Zhou, Learning effective brain connectivity with dynamic bayesian networks, *NeuroImage* 37 (3) (2007) 749 – 760. doi:http://dx.doi.org/10.1016/j.neuroimage.2007.06.003.
- [22] D. Posthuma, E. J. de Geus, E. J. Mulder, D. J. Smit, D. I. Boomsma, C. J. Stam, Genetic components of functional connectivity in the brain: The heritability of synchronization likelihood, *Human Brain Mapping* 26 (3) (2005) 191–198. doi:10.1002/hbm.20156.
URL <http://dx.doi.org/10.1002/hbm.20156>
- [23] D. Yu, U. Parlitz, Estimating parameters by autosynchronization with dynamics restrictions, *Phys. Rev. E* 77 (6) (2008) 066221.
- [24] M. Timme, Revealing network connectivity from response dynamics, *Phys. Rev. Lett.* 98 (22) (2007) 224101.
- [25] S. G. Shandilya, M. Timme, Inferring network topology from complex dynamics, *New Journal of Physics* 13 (1) (2011) 013004.

- [26] Z. Levnajić, A. Pikovsky, Network reconstruction from random phase resetting, *Phys. Rev. Lett.* 107 (3) (2011) 034101.
- [27] L. Prignano, A. Diaz-Guilera, Extracting topological features from dynamical measures in networks of kuramoto oscillators, *Phys. Rev. E* 85 (3) (2012) 036112.
- [28] L. L. T. Zhou, Link prediction in complex networks: A survey, *Physica A: Statistical Mechanics and its Applications* 390 (6) (2011) 1150 – 1170. doi:<http://dx.doi.org/10.1016/j.physa.2010.11.027>.
- [29] D. Yu, M. Righero, L. Kocarev, Estimating topology of networks, *Phys. Rev. Lett.* 97 (18) (2006) 188701.
- [30] Y. Wang, D. Chakrabarti, C. Wang, C. Faloutsos, Epidemic spreading in real networks: An eigenvalue viewpoint, in: *In Proc. of the 22nd International Symposium on Reliable Distributed Systems - IEEE SRDS'03, 2003*, pp. 25–34.
- [31] D. Chakrabarti, Y. Wang, C. Wang, J. Leskovec, C. Faloutsos, Epidemic thresholds in real networks, *ACM Trans. Inf. Syst. Secur.* 10 (4) (2008) 13:1 – 13:26.
- [32] R. Parshani, S. Carmi, S. Havlin, Epidemic threshold for the susceptible-infectious-susceptible model on random networks, *Phys. Rev. Lett.* 104 (25) (2010) 258701.
- [33] A. J. Ganesh, L. Massoulié, D. F. Towsley, The effect of network topology on the spread of epidemics, in: *In Proc. IEEE Infocom, Vol. 2, 2005*, pp. 1455–1466.
- [34] M. Draief, A. Ganesh, L. Massoulié, Thresholds for virus spread on networks, *Ann. Appl. Probab.* 18 (2) (2008) 359 – 378.
- [35] P. Van Mieghem, J. Omic, R. Kooij, Virus spread in networks, *IEEE/ACM Trans. Netw.* 17 (1) (2009) 1 – 14.
- [36] S. Gómez, A. Arenas, J. Borge-Holthoefer, S. Meloni, Y. Moreno, Discrete-time markov chain approach to contact-based disease spreading in complex networks, *Eur. Phys. Lett.* 89 (3) (2010) 38009.

- [37] S. Gómez, A. Arenas, J. Borge-Holthoefer, S. Meloni, Y. Moreno, Probabilistic framework for epidemic spreading in complex networks, *Int. J. Complex Systems in Science* 1 (2011) 47–51.
- [38] P. Holme, J. Saramäki, Temporal networks, *Physics Reports* 519 (3) (2012) 97–125.
- [39] R. K. Pan, J. Saramäki, Path lengths, correlations, and centrality in temporal networks, *Phys. Rev. E* 84 (2011) 016105. doi:10.1103/PhysRevE.84.016105.
- [40] J. L. Iribarren, E. Moro, Impact of human activity patterns on the dynamics of information diffusion, *Phys. Rev. Lett.* 103 (2009) 038702. doi:10.1103/PhysRevLett.103.038702.
- [41] R. Albert, A.-L. Barabási, Statistical mechanics of complex networks, *Reviews of Modern Physics* 74 (1) (2002) 47 – 97.
- [42] S. H. Watts, D. J. and Strogatz, Collective dynamics of 'small-world' networks, *Nature* 393 (6684) (1998) pp. 440442.
- [43] P. Erdős, A. Rényi, On the evolution of random graphs, *Publications of the Mathematical Institute of the Hungarian Academy of Sciences* 5 (1960) pp.17–61.

FREE DAMPED TORSIONAL VIBRATIONS OF CANTILEVER BEAM STRUCTURE

Abstract

In this paper are analysed free damped torsional vibrations of cantilever beam structure. The cross-section of the cantilever is of general shape with torsion bending constant equal to zero. It is with variable dimensions along the length of the beam. The material is assumed to be homogeneous and the coefficients of friction are constant and proportional to

I_p

The partial differential equation of dynamic equilibrium is derived by direct consideration of the elastic, inertia and damping torques that act on the infinitesimal line element. The resulting partial differential equation is linear with variable coefficients. The solution is seen into infinite convergent series. Each term of the series is product of eigen functions with respect to time, and space variable along the length of the beam. The eigen function with space is orthogonal and this property is used to isolate the time dependent eigen function.

This solution of the characteristic equation for the time eigen function is solved into close form including the structural friction. For this reason two nonlinear algebraic equations are derived to yield the damping coefficient and damped natural frequency for each mode of vibration. After solving for the time eigen function the general solution is used to derive an expression for the torque at any time and any cross-section.

Finally a particular case of free damped oscillations is considered, in order to demonstrate the estimate of the constants of integration. This is the case of initial torsional deformations at rest.

Key wards: torque, coefficient, friction, deformation, viscous, structural, damping

Introduction

In the theoretical and applied mechanics important place take the free damped oscillations of linear elastic structures. In practice the tendency is to analyse as simple as possible linear structures in order to get an estimate for the dynamical behavior. This dynamical behavior which characterises the structure will further find applications in practice. In this paper are analysed free damped torsional vibrations of a cantilever beam structure with variable cross-section along the length.

Up to now, lot of work has been done in the analysis with constant cross-section along the length, by the normal mode method [2, 3, 4]. But so far, no research is done with variable cross-section by the normal mode method. Hence, the investigations in this paper represent new research work in this field of applied mechanics.

The analysis is carried out under the action of structural and viscous damping. The structural damping arise from the internal friction of the material. The viscous damping is due to the friction and pressure distribution of the surrounding fluid, air or viscous fluid.

The derived partial differential equation of dynamic equilibrium is solved by making use of orthogonal functions, with respect to the space variable along the length [6]. The results obtained by the analysis are damped natural frequencies of vibration and orthogonal modes of vibrations.

These values are important for forced vibrations, to avoid resonance. The theoretical investigations in this paper have great applications in Aeronautical, Civil and Mechanical engineering and corresponding industries.

NOTATION

$[M]$	- mass matrix
$[k]$	- stiffness matrix
A_n	- constant of integration
a_m	- constant
C_n	- constant
c	- coefficient of structural damping
C_v	- coefficient of viscous damping
D_n	- constant

G	- elastic shear modulus
I_p	- second polar moment of inertia
J	- torsional constant
m	- integer index
n	- integer index
i	- integer index
j	- integer index
k	- integer index
l	- cantilever span
x	- space variable
t	- time variable
Q	- twisting torque
T_n	- time function
Y_n	- space function
η	- damping coefficient
ξ	- non dimensional viscous damping coefficient
μ	- structural damping coefficient
θ	- torsional deformation
ω_n	- undamped natural circular frequency
Ω_n	- damped natural circular frequency
λ	- complex constant
ρ	- material density

1.0 THEORETICAL ANALYSIS

The theoretical derivations in this paper for torsional vibrations of the cantilever beam

structure will be based on the assumptions made in the field of strength of materials and rigid body mechanics. Therefore we shall go straight into derivation of the partial differential equation of dynamic equilibrium of torques. For this reason let us consider the equilibrium of torques that act on a differential element of the structure. The structure is assumed to be of variable cross-sectional dimension along the length. In figure 1 are indicated the torques that act on the infinitesimal element.

Fig. 1 Components of torques acting on the infinitesimal element of the structure

From the theory of strength of materials and rigid body mechanics, the torques indicated in figure 1 are given by the following expressions.

Elastic and structural damping torques are given by:

$$Q = GJ \left(\frac{\partial \theta}{\partial x} + c \frac{\partial^2 \theta}{\partial t \partial x} \right)$$

..... a)

Inertia torque due to acceleration is:

$$Q_i = \rho I_\rho \frac{\partial^2 \theta}{\partial t^2} dx$$

..... b)

The viscous damping torque is given by:

$$Q_v = C_v \frac{\partial \theta}{\partial t} dx$$

..... c)

Considering the equilibrium of torques acting on the infinitesimal element, the following equation can be written:

$$-Q - Q_i - Q_v + Q + \frac{\partial Q}{\partial x} dx = 0$$

After clearing the above relation, follows the equation of dynamic equilibrium:

$$-Q_i - Q_v + \frac{\partial Q}{\partial x} dx = 0$$

..... 1)

Substituting the relevant expressions for the indicated members from relation a), b) and c) into above equation, yields:

$$-\rho I_\rho \frac{\partial^2 \theta}{\partial t^2} dx - C_v \frac{\partial \theta}{\partial t} dx + \frac{\partial}{\partial x} \left[GJ \left(\frac{\partial \theta}{\partial x} + c \frac{\partial^2 \theta}{\partial t \partial x} \right) \right] dx = 0$$

After canceling the infinitesimal length dx , the partial differential equation of dynamic equilibrium for torsional vibration is obtained in the form:

$$\rho I_{\rho} \frac{\partial^2 \theta}{\partial t^2} + C_v \frac{\partial \theta}{\partial t} - \frac{\partial}{\partial x} \left[GJ \left(\frac{\partial \theta}{\partial x} + c \frac{\partial^2 \theta}{\partial t \partial x} \right) \right] = 0$$

..... (2)

The above partial differential equation is linear, with variable coefficients, which will be solved by the normal mode method.

Let us assume that the solution of the above partial differential equation can be given by series of the form:

$$\theta_{(x,t)} = \sum_{n=1}^{n=k} T_{n(t)} Y_{n(x)}$$

..... (3)

In the above expression $T_{n(t)}$ is eigen function of time variable t only. Its form will be determined from the solution of equation (2). The space function

$$Y_{n(x)} = \sum_{m=1}^{m=k} b_{nm} \sin \left[\frac{(2m-1)\pi x}{2l} \right]$$

..... (4)

will be recognised as linear combination of the normal functions, solution to the undamped problem of torsional vibration, with constant cross-sectional area along the length. It satisfies the required boundary conditions imposed on it, which are:

$$Y_{n(0)} = 0$$

..... (4a)

$$\left(\frac{dY_n}{dx}\right)_{x=l} = 0$$

..... (4b)

Also, it represents approximate solution to our case of variable cross-sectional area, therefore I_p and J , along the length, with or without damping.

To solve for the eigen function $T_{n(t)}$ it is required to obtain exact or approximate values for the undamped natural frequencies of vibration. This paper will be concerned with approximate values for them.

For determining the undamped frequencies Rayleigh - Ritz method will be used [1,8].

The series representation for the solution to the problem of undamped natural frequencies and space functions is of the form (4), i.e.

$$\theta_{(x)} = \sum_{m=1}^{m=k} a_m \sin\left[\frac{(2m-1)\pi x}{2l}\right]$$

..... (5)

Applying the Rayleigh - Ritz method with this assumption for the mode shape, leads to the standard undamped eigen value and vectors problem:

$$(-\omega^2[M] + [k])\{a\} = \{0\}$$

..... (6)

In the above matrix equation $[M]$ and $[k]$ are symmetric square matrices, and ω and $\{a\}$ are the undamped natural frequencies and modes of vibrations [5]. The elements of these matrices are given by the relations:

$$k_{ij} = \frac{(2i-1)(2j-1)\pi^2}{4l^2} G \int_0^l J \cos\left[\frac{(2i-1)\pi}{2l}x\right] \cos\left[\frac{(2j-1)\pi}{2l}x\right] dx$$

..... (7a)

$$m_{ij} = \rho \int_0^l I_p \sin \left[\frac{(2i-1)\pi}{2l} x \right] \sin \left[\frac{(2j-1)\pi}{2l} x \right] dx$$

..... (7b)

The solution of the eigen value problem (6) gives the required undamped natural frequencies k and vectors of modes k . Each of these vectors $\{a\}_n$ represent the vector of constants $\{b\}_n$ for the function $Y_{n(x)}$ given by expression (4).

Now, anyone of these functions $Y_{n(x)}$ satisfies approximately the relation:

$$\frac{d}{dx} \left[GJ \frac{dY_n}{dx} \right] \cong -\omega_n^2 Y_{n(x)} \rho I_p$$

..... (8)

The above relation follows directly from the definition of the $Y_{n(x)}$. If the $Y_{n(x)}$ were the exact solution of the undamped problem of vibration, the equal sign must hold in equation (8).

The normal mode function are orthogonal-normal functions with respect to each other, this is:

$$\int_0^l I_p Y_{n(x)} Y_{m(x)} dx = 0 \quad n \neq m$$

..... (9)

The polar moment of inertia I_p is the weight function. Now, the above important relationship will be proved.

To prove relationship (9) we will consider the product

$$Y_{n(x)} Y_{m(x)} = [b]_n \{ \sin \beta_n x \} [\sin \beta_m x] \{ b \}_m$$

..... (10)

where:

$[b]_n$ - eigen vector row

$\{\sin \beta_n x\}$ - column of normal functions

$$\beta_n = \frac{(2n-1)\pi}{2l} \quad n = 1, 2, 3, \dots, k$$

$\{b\}_m$ - eigen vector column

$[\sin \beta_m x]$ - row of normal function

$$\beta_m = \frac{(2m-1)\pi}{2l} \quad m = 1, 2, 3, \dots, k$$

Multiplying equation (10) by I_p and integrating from 0 to l , leads to the following relation:

$$\int_0^l I_p Y_{n(x)} Y_{m(x)} dx = \frac{1}{\rho} [b]_n [M] \{b\}_m$$

..... (11)

In the above expression the matrix $[M]$ is the symmetric mass matrix defined in equation (6).

But from theory of eigen value problem, we know that the eigen vectors of equation (6) are normal to each other, with respect to either $[M]$ or $[k]$ matrices. Hence the relation (11) states that (9) is satisfied. This concludes the proof of orthogonality of normal mode functions one to another with respect to I_p as a weighting function.

Now, differentiating the assumed solution (3) with respect to t and x , and substituting into equation (2) yields the following equation:

$$\sum_{m=1}^{m=k} \left[\rho I_p T''_{n(t)} Y_{n(x)} + C_v T'_{n(t)} Y_{n(x)} - T_{n(t)} \frac{d}{dx} \left(GJ \frac{dY_{n(x)}}{dx} \right) - c T'_{n(t)} \frac{d}{dx} \left(GJ \frac{dY_{n(x)}}{dx} \right) \right] = 0$$

Making use of relation (8) and assuming that $Y_{n(x)}$ is the exact solution of the undamped case, the above equation can be written in the following form:

$$\sum_{m=1}^{m=k} \left[\rho I_p T''_{n(t)} Y_{n(x)} + C_v T'_{n(t)} Y_{n(x)} + (\rho I_p \omega_n^2 T_{n(t)} + c \rho I_p \omega_n^2 T'_{n(t)}) Y_{n(x)} \right] = 0$$

..... (12)

Assuming that the coefficient of viscous damping C_v is proportional to the cross-sectional dimension along the length of the cantilever, it is possible to write that:

$$C_v = z I_p$$

..... (13)

Substituting the above value for C_v in the equation (12) and rearranging terms yields the following expression:

$$\sum_{n=1}^{n=k} \left[T''_{n(t)} + \frac{z}{\rho} T'_{n(t)} + c \omega_n^2 T'_{n(t)} + \omega_n^2 T_{n(t)} \right] \rho I_p Y_{n(x)} = 0$$

..... (14)

Multiplying the above series by $Y_{m(x)}$, $m = 1, 2, 3, \dots, n, \dots, k$, and integrating from 0 to l , using the orthogonality condition (9), isolates the k equations, each one of the form:

$$T''_{n(t)} + \frac{z}{\rho} T'_{n(t)} + c \omega_n^2 T'_{n(t)} + \omega_n^2 T_{n(t)} = 0$$

..... (15)

Let us introduce the following substitution:

$$\frac{z}{\rho} = 2\xi\omega_n$$

..... (16)

This means that the z constant takes different value for each mode of vibration. The ξ is other constant.

The coefficient of structural friction c is given by the formula:

$$c = \frac{\mu}{\Omega_n} \dots\dots\dots (17)$$

In the above expression the angular frequency Ω_n is the frequency of damped vibration, and μ is the coefficient of hysteretic damping. With the above expression for c , the differential equation for each $T_{n(t)}$ becomes:

The term in the bracket in the above equation is generated by viscous and structural damping.

The expected solution of equation (18) is damped vibration with respect to the independent variable t . Hence damped harmonic function can be seen as possible solution.

Let us assume that $T_{n(t)}$ can be given by the expression:

$$T_{n(t)} = C_n e^{\lambda_n t} \dots\dots\dots (19)$$

In the above expected solution C_n is constant of integration and in original form is complex number. The exponential constant λ_n is also complex number of the form:

$$\lambda_n = -\eta_n + i\Omega_n \dots\dots\dots (20)$$

The real part of this complex number is related to the damping of the oscillating motion. The imaginary part of the constant λ_n , Ω_n , is the angular frequency of vibration of the structure, which is in the term in bracket of equation (18).

The estimate of λ_n can be made by obtaining simultaneous solution to two nonlinear algebraic equations in η_n and Ω_n . These equations are obtained by equating the real and imaginary part to zero of the complex characteristic function:

$$\lambda_n^2 + 2\xi\omega_n\lambda_n + \mu\frac{\omega_n^2}{\Omega_n}\lambda_n + \omega_n^2 = 0$$

.....(21)

After some arithmetic on the above complex number, the two algebraic nonlinear equations are obtained in the following form:

$$-\Omega_n^3 + \Omega_n\eta_n^2 + \Omega_n\eta_n2\xi\omega_n + \eta_n\mu\omega_n^2 + \Omega_n\omega_n^2 = 0$$

.....(22)

$$\Omega_n2\xi\omega_n + 2\Omega_n\eta_n + \mu\omega_n^2 = 0$$

.....(23)

Having obtained the complex number λ_n the general solution for the time dependent function $T_{n(t)}$ as given by the expression (19), in developed form is:

$$T_{n(t)} = e^{-\eta_n t} (A_n \cos \Omega_n t + B_n \sin \Omega_n t)$$

.....(24)

The A_n and B_n are real constants of integration and are determined from the boundary conditions of the cantilever vibration.

Substituting expression (24) into expression (3) the general solution of the damped free torsional vibrations of the cantilever structure can be written in the form:

$$\theta_{(x,t)} = \sum_{n=1}^{n=k} e^{-\eta_n t} (A_n \cos \Omega_n t + B_n \sin \Omega_n t) Y_{n(x)}$$

.....(25)

The above solution is applicable to cantilever structure with cross-sections whose torsion-bending constant is equal to zero.

Now, the expression for torque at any time t and section x will be derived. For this purpose the derivative of the normal modes with respect to x is required. After differentiating $Y_{n(x)}$ w.r.t x yields:

$$\frac{dY_n}{dx} = \sum_{m=1}^{m=k} b_{nm} \frac{(2m-1)\pi}{2l} \cos\left[\frac{(2m-1)\pi}{2l}x\right]$$

..... (26)

Also, the time derivative of $T_{n(t)}$ w.r.t t we obtain:

$$\frac{dT_n}{dt} = e^{-\eta_n t} [(\Omega_n B_n - \eta_n A_n) \cos \Omega_n t - (\eta_n B_n + \Omega_n A_n) \sin \Omega_n t]$$

..... (27)

Inserting the relevant expressions from (26) and (27) into (a), the following formula for the torque is obtained:

$$Q_{(x,t)} = GJ \sum_{n=1}^{n=k} e^{-\eta_n t} \left\{ \left[A_n \left(1 - \mu \frac{\eta_n}{\Omega_n} \right) + \mu B_n \right] \cos \Omega_n t + \left[B_n \left(1 - \mu \frac{\eta_n}{\Omega_n} \right) - \mu A_n \right] \sin \Omega_n t \right\} \frac{dY_n}{dx}$$

.....
(28)

The formula (28) is very slow convergent series. The number k can be very large, depending on the initial boundary condition, cross-section, and of course the precision required. All it states is, that the problem of calculating the torque on a cantilever structure with free end and for free damped vibration is solvable in this manner.

2.0 Numerical Example

An numerical example is presented, in order to show the theoretical investigations given in this paper. The structure to be analysed is shown in figure 2. It is with variable rectangular cross-section, which dimensions vary along the length, as indicated in the figure. The material is steel, with hysteretic damping coefficient $\mu = 0.04$, modulus of elasticity $E = 2.1 * 10^{11} \text{ N / m}^2$ Poisson's

ratio $\nu = 0.3$, damping coefficient $\xi = 0.02$ and material density $\rho = 7850 \text{ kg} / \text{m}^3$

Fig. 2 Cantilever beam structure with variable cross-sectional dimensions, as shown.

According to the dimensions shown in fig. 2 a) , b) , c), the second polar moment of inertia of the cross-section at any section x is given by the formula.

$$I_{p(x)} = 6.6666667 * 10^{-7} * (1 - 0.5x)[0.16 + 0.04(1 - 0.5x)]$$

..... (2.1)

The torsion constant $J_{p(x)}$ is given by the expression [7]:

$$J_{p(x)} = 1.0666667 * 10^{-7} * (1 - 0.5x)^3 [1 - 0.315(1 - 0.5x)]$$

..... (2.2)

The cantilever is initially at rest, so that its initial velocity is zero, and initial displacement

$\theta_{0(x,t=0)}$ given by the expression

$$\theta_{0(x,t=0)} = \theta_0 \left(\frac{x}{l} \right)$$

..... (2.3)

From expression (27), since the velocity at time $t = 0$ is zero, follows that:

$$B_n = \left(\frac{\eta_n}{\Omega_n} \right) A_n$$

..... (2.4)

Therefore the response for $\theta_{(x,t)}$ will be given from formula (25) in the form:

$$\theta_{(x,t)} = \sum_{n=1}^{n=k} e^{-\eta_n t} A_n \left(\cos \Omega_n t + \frac{\eta_n}{\Omega_n} \sin \Omega_n t \right) Y_{n(x)}$$

..... (2.5)

Now remains to determine the constants A_n . This will be done by equating (2.3) and (2.5) for $t = 0$. Therefore it follows that:

$$\theta_0 \frac{x}{l} = \sum_{n=1}^{n=k} A_n Y_{n(x)}$$

..... (2.6)

Multiplying relation (2.6) by $I_{p(x)} Y_{n(x)}$ and integrating from 0 to l yields:

$$A_n = \frac{\theta_0}{l} \int_0^l x I_{p(x)} Y_{n(x)} dx \frac{1}{\Phi_n}$$

..... (2.7a)

Where Φ_n is given by the expression:

$$\Phi_n = \int_0^l I_{p(x)} Y_{p(x)}^2 dx$$

..... (2.7b)

For any particular section $x = r$, the solution (2.5) can be written as:

$$\theta_{(x=r,t)} = \sum_{n=1}^{n=k} T_{n(t)} D_{(x=r)}$$

..... (2.8)

For the cantilever shown in figure 2, for $x = r = l$ the following values for

$\eta_n, \Omega_n, D_n, n = 1, 2, 3, \dots, 10$ are obtained and tabulated in table 1.

TABLE I

Numerical results for relevant response values

n	η_n 1/sec	Ω_n rad/sec	D_n / θ_0
1	-2101.311	53469.827	0.000964
2	-2502.922	62498.052	0.000170
3	-1817.959	45394.500	0.001139
4	-1564.974	39067.845	0.003648
5	-154.974	3869.718	0.924878
6	-373.583	9328.373	-0.008691
7	-607.675	15171.162	0.037934
8	-1083.201	27047.581	0.009759
9	-1322.721	33028.385	0.003117
10	-844.922	21097.726	0.006125

3.0 CONCLUSION

From the theoretical investigation in this paper it is possible to draw the following conclusions.

Firstly, for the torsional vibrations of a cantilever beam structure it can be concluded that the

modal function are normal to each other with weight function I_p . This statement was proved in the analysis. Secondly, the modeling of the viscous damping coefficient, equation (16), together with the two nonlinear algebraic equation (22), and (23) is physically correct. Namely it allows calculation of almost constant modal damping parameter for each independent mode of vibration.

These two conclusions bring the new theoretical achievements in this paper.

Резиме

Во овој труд се анализирани слободни придушени торзиони вибрации на конзол-на конструкција. Попречниот пресек на конзолната конструкција е сосема произволен, со торзиона-свиткувачка константа еднаква на нула. Тој е со променливи димензии долж распонот на конзолата. Материјалот од кој е направена конструкцијата е хомоген.

Изведена е парцијална диференцијана равенка на динамичка рамнотежа со директно разгледување на еластичниот, инерцијалниот и придушувачките спрегови кои дејствуваат на бесконечно мал елемент. Така изведената диференцијална равенка е линеар-на со променливи коефициенти. Нејзиното решение е претпоставено со конвергентен ред. Секој член на овој ред претставува производ на сопствени функции од време и простор.

Карактеристичната равенка на сопствената функција по време е решена во затворена форма, вклучувајќи го и конструктивното придушување. Така да изведени се две нелинеарни алгебарски равенки, чие решение ги дава коефициентот на придушување и кружната придушна фреквенција, за секоја форма на осцилирање. Добиеното општо решение е употребено да се изведе израз за торзиониот спрег во било кое време и било кој пресек.

На крајот е разгледуван посебен случај од интерес на слободни придушни осцилации, со цел да се демонстрира одредувањето на константите на интеграција. Ова е (сторено) направено за случајот на конзола со почетни деформации во мирување.

REFERENCES

- [1] HURTY, C.W., RUBINSTEIN F.M.; Dynamics of STRUCTURES PRENTICE – HALL, Inc. ENGLEWOOD CLIFFS, NEW JERSEY 1964 P.193-161
- [2] МИHAJLOV, M.; DEFORMATIONS OF THINWALLED I SECTION DUE TO TORQUE AND AXIAL STRESSES, CONTRIBUTIONS, SEC. MATH. TECH. Sci., MANU X 1-2 (1989) Offprint
- [3] МИHAJLOV, M.; NATURAL MODES OF FREE UNDAPEDED TORSIONAL VIBRATIONS FOR SIMPLE BEAM STRUCTURE, CONTRIBUTIONS, SEC. MACH. TECH. Sci., MANU XIII 1 (1992) Offprint

[4] MIHAJLOV. M., NATURAL MODES OF UNDAMPED TORSIONAL VIBRATIONS FOR A CANTILEVER STRUCTURE WITH PARTICULAR BOUNDARY CONDITIONS, CONTRIBUTIONS Sec. MATH. TECH. Sci., MANU XV 2 (1994) Offprint

[5] MIHAJLOV. M.; ELEMENT LOCAL VIBRATIONS INFLUENCE ON THE OVERALL EIGEN VALUES AND VECTORS FOR TRUSS – FRAMES WITH RIGID JOINTS., MACEDONIAN ASSOCIATION OF THE STRUCTURAL ENGINEERS , 7th INTERNATIONAL SYMPOSIUM, OHRID 1997 VOLUME 3, CT 47/1 - 6

[6] MIHAJLOV. M.; CONTRIBUTION TO THE ANALYSIS OF AXIAL VIBRATIONS, PROCEEDINGS, 6 – th SYMPOSIUM ON THEORETICAL AND APPLIED MECHANICS, STRUGA 1998 P 249 – 255 VOLUME 1

[7] SERAFIMOV,P.; OSNOVI NA TEORIJA NA KONSTRUKCIITE I OPREDELENI NOSA□, MAKEDONSKA KNIGA, SKOPJE 1984 P 325 – 353

[8] WARBURTON, G.B.; THE DYNAMICAL BEHAVIOUR OF STRUCTURES, PERGAMON PRESS, OXFORD 1976 P 115 – 151

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TABLE I - Dr. Mile R. Mihajlov

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1	-2101.311	53469.827	0.000964
2	-2502.922	62498.052	0.000170
3	-1817.959	45394.500	0.001139
4	-1564.974	39067.845	0.003648
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9	-1322.721	33028.385	0.003117
10	-844.922	21097.726	0.006125

Dr. Mile R. Mihajlov
Potpis na slikite

Fig. 1 Components of torques acting on the
Infinitesimal element of the structure

Fig. 2 Cantilever beam structure with variable
cross-sectional dimension, as shown

Dr. Mile R. Mihajlov

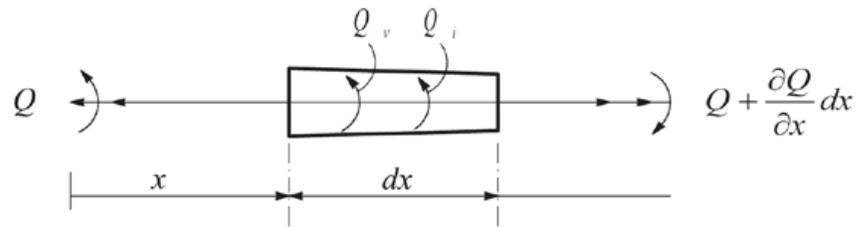


Fig. 1

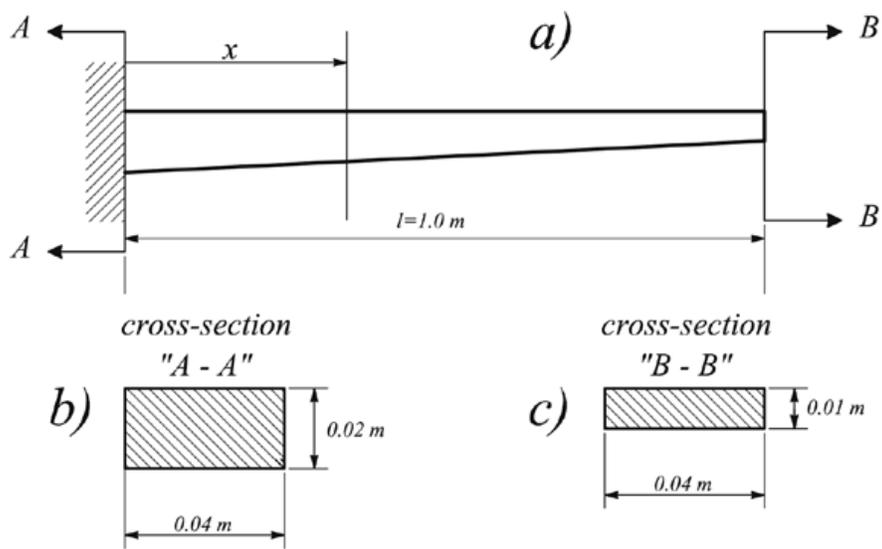


Fig. 2

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