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Comparison Between Exact and Quasi-Static Methods for HF Analysis of Horizontal Buried Wires

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***Abstract*—The validity domain of the quasi-static method for computation of high frequency and transient characteristics of horizontal buried wires can be established by comparison with an exact analytical method. Usually, limitations of the quasi-static method are derived for practical characteristics, but these are strongly dependent on the specific case and computed quantities. This paper presents an analysis of the differences in the application of the exact and quasi-static Green's function in a method of moments approach for two important cases: distribution of currents in directly fed wires and induced currents in passive wires. It is concluded that the validity domains of the quasi-static method in these two cases are very different.**

***Index Terms*—Circuit modeling, Green's functions, grounding, lightning, method of moments (MoM), transient analysis, transmission line theory.**

I. Introduction

High frequency (HF) and transient analysis related to lightning, faults, or other electromagnetic interferences in buried conductors or networks of conductors that are part of power, telecommunication, or railway systems is of interest in electromagnetic compatibility (EMC) studies [1]. Classical modeling approaches are based on circuit theory with lumped [2] or distributed parameters [3], which is also the case in many modern approaches, e.g., [1], [4], and [5]. Since circuit theory approaches are based on the quasi-static approximation, their validity is limited to a certain upper frequency [1]. On the other hand, full-wave methods have been recently introduced, e.g., [6] and [7], based on the solution of the Maxwell's equations by the method of moments (MoM) [8]. However, since electromagnetic MoM models are based on an exact mathematical solution by Sommerfeld [9], they might serve as a standard for comparison of more approximate models [10].

The validity domain of the circuit theory approaches has been recently studied in [10] and [11]; however, considering different metrics for comparison, Olsen and Willis [10] consider the touch and step voltages in the frequency domain, and Theethayi *et al.* [11] consider the transient currents and voltages in the time domain (both for directly fed wires). Both studies [10], [11] suggest different limits of the validity domain of the considered circuit theory approaches. However, although both studies give a direct insight into some practical characteristics, conclusions are related to the specific choice of the metrics, the system under study, and the methodology of the solution of the complex mathematical models. Another recent publication [12, p. 335] also considers "the classical transmission line approach to be relevant for practical use" for coupling to buried wires. It is, therefore, of interest to investigate more thoroughly the validity domain of the circuit approaches.

Classical circuit models with both distributed and lumped parameters are based on the quasi-static approximation. As a first step toward a better understanding of their limitations, we look at the most basic case of the horizontal elemental electric dipole in a conducting half-space, for which there is a full-wave exact solution [13]. When the Green's function for this elementary dipole is obtained, the familiar solution to the problem of extended wires involves integrating over sources using Green's theorem [8]. In this paper, we analyze differences in the application of the exact and quasi-static Green's function in a MoM-based electromagnetic model [11] for two important cases: distribution of currents in directly fed wires and induced currents in passive wires. In both cases, we consider bare wires.

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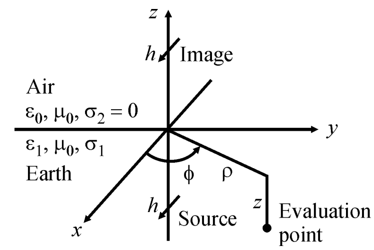

Fig. 1. Coordinates for evaluation of fields caused by buried source.

II. Full-Wave Solution

Sommerfeld [9] first published the exact solution of the electromagnetic field for an electric dipole near an interface. The geometry of the problem considered is illustrated in Fig. 1. The horizontal electric dipole is in the direction of the *x*-axis. The dipole and the field evaluation point are both below the boundary between the air and the earth. The designations of the coordinates and the characteristics of the earth and the air are given in Fig. 1.

We consider a dipole with harmonic current moment $p = I\ell$ with angular frequency ω. The time variation exp($j\omega t$) is suppressed. The wavenumbers of the earth k_1 and the air k_2 are

$$k_1^2 = \omega^2\mu_0\left(\varepsilon_1 - \frac{j\sigma_1}{\omega}\right) \qquad k_2^2 = \omega^2\mu_0\varepsilon_0 \tag{1}$$

The complete set of field equations in cylindrical coordinates from Banos [13] is given here for reference

$$\begin{aligned} E_\rho &= \frac{-pj\omega\mu_0}{4\pi k_1^2}\cos\phi\left[\frac{\partial^2}{\partial\rho^2}\left(g_1 - g_2 + k_1^2 V_1\right) + k_1^2\left(g_1 - g_2 + U_1\right)\right] \\ E_\phi &= \frac{pj\omega\mu_0}{4\pi k_1^2}\sin\phi\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(g_1 - g_2 + k_1^2 V_1\right) + k_1^2\left(g_1 - g_2 + U_1\right)\right] \\ E_z &= \frac{-pj\omega\mu_0}{4\pi k_1^2}\cos\phi\left[\frac{\partial^2}{\partial\rho\partial z}\left(g_1 + g_2 - k_2^2 V_1\right)\right] \end{aligned} \tag{2}$$

$$\begin{aligned} H_\rho &= \frac{p\sin\phi}{4\pi}\left[\frac{\partial}{\partial z}\left(g_1 - g_2 + U_1\right) - \frac{1}{\rho}\frac{\partial W_1}{\partial\rho}\right] \\ H_\phi &= \frac{p\sin\phi}{4\pi}\left[\frac{\partial}{\partial z}\left(g_1 - g_2 + U_1\right) - \frac{\partial^2 W_1}{\partial\rho^2}\right] \\ H_z &= \frac{-p\sin\phi}{4\pi}\left[\frac{\partial}{\partial\rho}\left(g_1 - g_2 + U_1\right)\right] \end{aligned} \tag{3}$$

where

$$V_1 = 2\int_0^\infty \frac{\exp\left[\gamma_1\left(h - z\right)\right]}{k_1^2\gamma_2 + k_2^2\gamma_1} J_0\left(\lambda\rho\right) d\lambda$$

$$U_1 = 2\int_0^\infty \frac{\exp\left[\gamma_1\left(h - z\right)\right]}{\gamma_1 + \gamma_2} J_0\left(\lambda\rho\right) d\lambda$$

$$W_1 = 2\int_0^{\infty} \frac{\gamma_2 - \gamma_1}{k_1^2\gamma_2 + k_2^2\gamma_1} \exp\left[\gamma_1 (h - z)\right] J_0(\lambda\rho)\, d\lambda$$

$$\gamma_1 = \left(\lambda^2 - k_1^2\right)^{1/2}, \qquad \gamma_2 = \left(\lambda^2 - k_2^2\right)^{1/2} \tag{4}$$

$$g_1 = \frac{\exp(-jk_1 r_1)}{r_1}, \qquad r_1 = \left[\rho^2 + (h+z)^2\right]^{1/2}$$

$$g_2 = \frac{\exp(-jk_1 r_2)}{r_2}, \qquad r_2 = \left[\rho^2 + (h-z)^2\right]^{1/2} \tag{5}$$

and $J_0(\cdot)$ is the Bessel function of the first kind and zero order. V_1, U_1, and W_1 (4) are known as Sommerfeld integrals. g_1 and g_2 (5) are related to the fields of the dipole in the position of the source and the image, respectively, in the unbounded medium with the characteristics of earth.

III. Quasi-Static Solution

The electric and magnetic fields for the dc horizontal electric dipole were derived by Banos and Wesley [14]. The quasi-static forms follow from the requirement $|k_1 r_1| \ll 1$, with $\omega \to 0$, but without requiring $\omega = 0$ [15]. The static equations from [14] can be simply extended for the quasi-static case by substituting σ_1 with $(\sigma_1 + j\omega\varepsilon_1)$ [15]. The quasi-static field components are

$$E_\rho = \frac{p\cos\phi}{4\pi(\sigma_1 + j\omega\varepsilon_1)} \frac{\partial^2}{\partial\rho^2}\left[\frac{1}{r_1} + \alpha_1\frac{1}{r_2}\right]$$

$$E_\phi = -\frac{p\sin\phi}{4\pi(\sigma_1 + j\omega\varepsilon_1)} \frac{1}{\rho}\frac{\partial}{\partial r}\left[\frac{1}{r_1} + \alpha_1\frac{1}{r_2}\right]$$

$$E_z = \frac{p\cos\phi}{4\pi(\sigma_1 + j\omega\varepsilon_1)} \frac{\partial^2}{\partial z\partial\rho}\left[\frac{1}{r_1} + \alpha_1\frac{1}{r_2}\right] \tag{6}$$

$$H_\rho = \frac{p\sin\phi}{4\pi}\frac{\partial}{\partial z}\left[\frac{1}{r_1} + \alpha_1\frac{r_2 + (z-h)}{\rho^2}\right]$$

$$H_\phi = \frac{p\cos\phi}{4\pi}\frac{\partial}{\partial z}\left\{\frac{1}{r_1} + \alpha_1\left[\frac{1}{r_2} - \frac{r_2 + (z-h)}{\rho^2}\right]\right\}$$

$$H_z = -\frac{p\sin\phi}{4\pi}\frac{\partial}{\partial\rho}\frac{1}{r_1}. \tag{7}$$

Here,

$$\alpha_1 = \frac{\sigma_1 + j\omega(\varepsilon_1 - \varepsilon_0)}{\sigma_1 + j\omega(\varepsilon_1 + \varepsilon_0)} \tag{8}$$

is a coefficient of the quasi-static image method [16].

The quasi-static electric field (6) is determined by the image method, but not the quasti-static magnetic field (7), which agrees with Wait's arguments [17]. The extent of simplification of the full-wave solution (2)–(5) by adopting quasi-static approximation (6)–(8) is evident by inspection. Many components that define complex field structure are disregarded in the quasi-static solution. It is, therefore, of interest to determine the validity domain of the simplified solution. However, it is not a simple task, since such a validity domain strongly depends on the situation and the computed quantities.

IV. Limitations of the Quasi-Static Solution

The basic limitations for applying the quasi-static solution (6)–(8) follow from the requirement $|k_1 r_1| \ll 1$, for example, $|k_1 r_1| \le 0.1$. However, such limitations can be an order of magnitude larger for quantities computed by the MoM [10], which follows from the stationary property. Thus, a frequently used limitation requires r_1 being smaller than one-tenth of the wavelength λ in the medium [8]. Both limitations

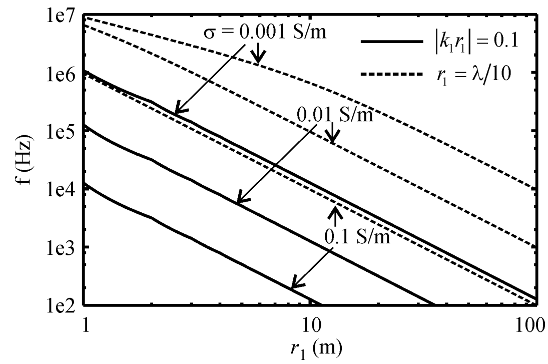

Fig. 2. Limitations of the quasi-static solution.

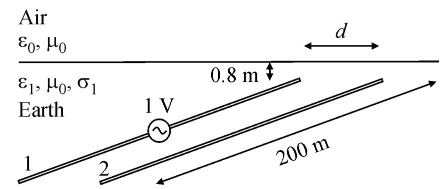

Fig. 3. Geometry of the test case for comparison studies.

are illustrated in Fig. 2, which shows frequencies f and distances r_1 for which both limitations, that is, $|k_1 r_1| = 0.1$ and $r_1 = \lambda/10$ are fulfilled for different values of the conductivity σ of the earth.

Nevertheless, these limitations are not widely applicable to practical cases, which will be demonstrated in the example given in the next section.

V. Comparison Between Exact and Quasi-Static Solutions

To compare solutions based on the exact (3) and quasi-static (6) Green's functions, we compare currents along two horizontal buried bare wires solved by applying the MoM [7]. The geometry of the test case is illustrated in Fig. 3.

The wires are mutually parallel and are denoted by "1" and "2," respectively. Their length is 200 m, the radius is 7 mm, the depth of burial is 0.8 m, and the distance d between them is 10 m. A harmonic voltage source with 1-V amplitude is located in the middle point of wire "1." All conductors' ends are left "open," therefore, due to the MoM solution [8], the longitudinal current is forced to be zero there.

Fig. 4 shows the magnitude of the longitudinal currents along the wires (the abscissa x is the distance along the wire from the wire's end). The conductivity of the earth in all cases is 0.01 S/m and the relative permittivity is 10. The frequency of the source in Fig. 4(a) is 100 kHz, in Fig. 4(b) is 1 MHz, and in Fig. 4(c) is 10 MHz, respectively. First, the wire "1" (see Fig. 3) is considered alone (without the wire "2"), and "1" in Fig. 4 denotes the computed currents along the wire. Then, the wire "2" (see Fig. 3) is added, and "2" in Fig. 4 denotes the computed induced currents along the wire.

The first conclusion from the results shown in Fig. 4 is that there is a low-frequency range in which the "exact" and quasi-static solution might lead to "acceptable" agreement, for both cases, such as in Fig. 4(a) for $f = 100$ kHz. The wavelength λ in earth for this case is about 71 m, and the differences between the currents obtained by the models might be "acceptable" for distances between points along the wires and the voltage source similar to λ. The quasi-static solution overestimates the induced currents in wire "2"; however, for distances

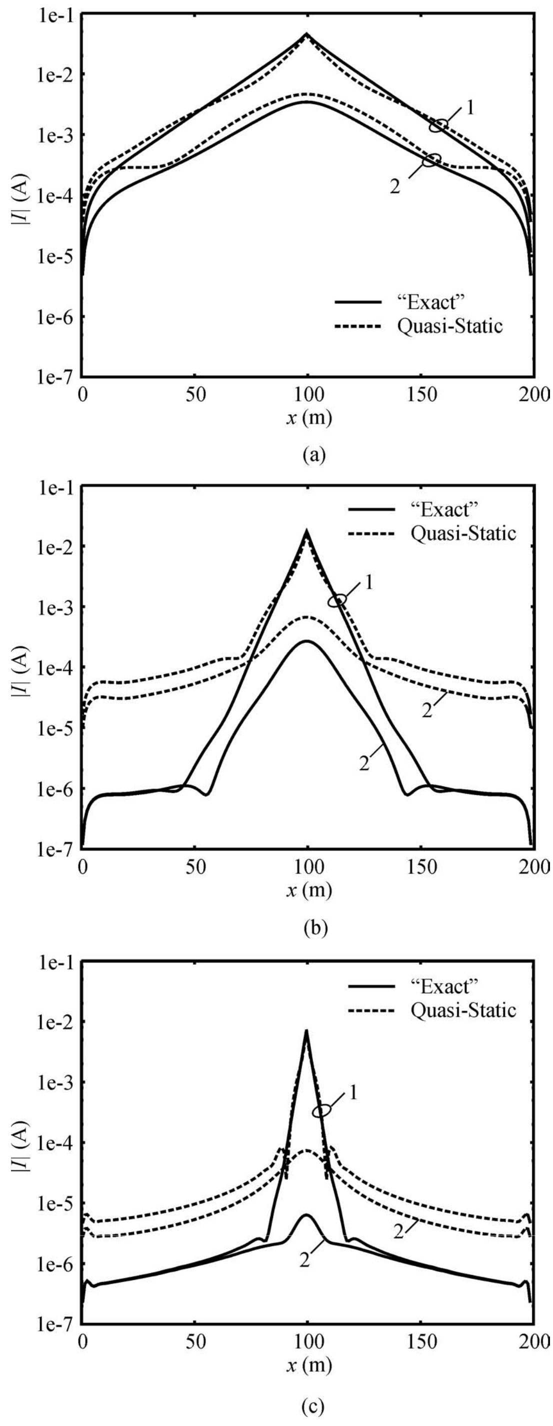

Fig. 4. Magnitude of the harmonic currents along the wires "1" and "2" (see Fig. 3) computed by the "exact" and quasi-static approaches (earth conductivity is 0.01 S/m and relative permittivity is 10). Results shown for wire "1" do not consider the presence of wire "2." (a) $f = 100$ kHz. (b) $f = 1$ MHz. (c) $f = 10$ MHz.

from the source, which are larger than λ, i.e., at the ends of the wire, such an overestimate is larger.

The picture is quite different for the higher frequencies considered. In Fig. 4(b), the frequency f is 1 MHz and the wavelength λ in earth is about 22 m. Computed currents along wire "1" for a distance of about λ from the source in both directions are in good agreement for both "exact" and quasi-static methods. Moreover, the differences become very large, that is, the quasi-static method largely overestimates the intensity of the current. However, it is important to note that the intensities of the currents diminish by several orders of magnitude as compared to near the source. The case of induced currents in wire "2" is different; there is no agreement of the results anywhere along the wire.

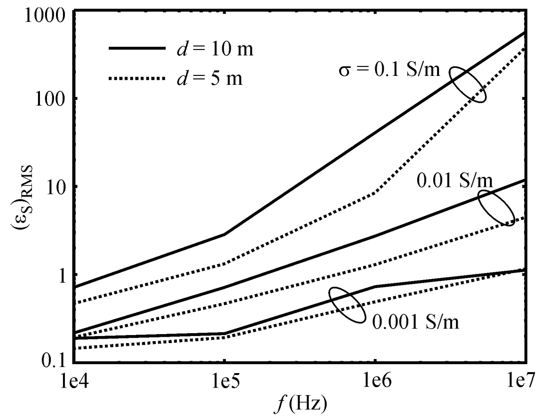

Fig. 5. Normalized rms error of induced currents in wire "2" computed by the quasi-static method (d is the distance between the wires, Fig. 3).

The conclusions are similar even for $f = 10$ MHz in Fig. 4(c). Here $\lambda = 6.6$ m, and currents computed by the two methods along wire "1" at such distance from the source are in good agreement. Similar to the previous case, there is no agreement between the two methods for computed currents at larger distances along wire "1" and all along wire "2."

If one considers the power flow in the case of the directly fed wire "1," for example, the power discharged to the earth, it is obvious that most of the power is discharged near the source, since the currents diminish faster with distance from the source. Therefore, it is possible to compute quantities that are related to the feed point, such as the input impedance, by the quasi-static method. However, this is not the case with the currents further away from the source and the induced currents in nearby conductors. It is worth noting that the usual limits for validity of the quasi-static method, as mentioned in Section IV, are not applicable in any case considered in Fig. 4.

In a comparison of different methods for computation of induced currents, in addition to the detailed comparison of current distributions, the following scalar parameter, referred to as normalized rms error, has been used [18]:

$$(\varepsilon_S)_{\mathrm{rms}} = \left[\frac{\sum_{i=1}^{N} \left| \underline{I}_{Ei} - \underline{I}_{\mathrm{QS}i} \right|^2}{\sum_{i=1}^{N} \left| \underline{I}_{Ei} \right|^2} \right]^{1/2} \tag{9}$$

where $\underline{I}_{Ei}$ and $\underline{I}_{\mathrm{QS}i}$ are complex current coefficients computed by the MoM using the "exact" and quasi-static Green's function, respectively, and N is number of segments.

Fig. 5 shows the normalized rms error (9) of induced currents in wire "2" computed by the quasi-static method with the "exact" method as a reference, for the test case considered with different earth conductivities, and for two distances between wires, $d = 10$ m and $d = 5$ m. The number of segments N used in the computations was large enough for the convergence of the results. It is obvious that the validity domain of the quasi-static method might be confined to the low-frequency range, and therefore, this method is not always suitable for computations of HF or fast-transient induced currents in nearby conductors. This also asserts a practical limit on the use of this method in cases of more complex conductor arrangements when induced currents between different conductors are important for the computed quantities.

VI. Conclusion

The validity domain of the quasi-static method in the computations of HF and transient characteristics of horizontal buried bare wires strongly depends on the case considered and the computed quantity.

There is a large difference between the validity domains of the quasi-static method for computation of currents near the feed point of directly fed wires and induced currents in nearby wires. The best agreement between the results of the quasi-static and the "exact" methods, in the cases considered, was for currents near the feed point, at distances less than the wavelength in the earth. However, the errors were large for computed induced currents in a nearby passive conductor, larger for better earth conductivity and for larger distance.

References

[1] F. M. Tesche, M. Ianoz, and T. Karlsson, *EMC Analysis Methods and Computational Models*. New York: Wiley, 1997.

[2] R. Rudenberg, *Electrical Shock Waves in Power Systems*. Cambridge, MA: Harvard Univ. Press, 1968.

[3] E. D. Sunde, *Earth Conduction Effects in Transmission Systems*, 2nd ed. New York: Dover, 1968.

[4] A. Geri, "Behavior of grounding systems excited by high impulse currents: The model and its validation," *IEEE Trans. Power Del.*, vol. 14, no. 3, pp. 1008–1017, Jul. 1999.

[5] Y. Liu, M. Zitnik, and R. Thottappillil, "An improved transmission-line model of grounding system," *IEEE Trans. Electromagn. Compat.*, vol. 43, no. 3, pp. 348–355, Aug. 2001.

[6] G. J. Burke and E. K. Miller, "Modeling antennas near to and penetrating a lossy interface," *IEEE Trans. Antennas Propag.*, vol. 32, no. 10, pp. 1040–1049, Oct. 1984.

[7] L. Grcev and F. Dawalibi, "An electromagnetic model for transients in grounding systems," *IEEE Trans. Power Del.*, vol. 5, no. 4, pp. 1773–1781, Oct. 1990.

[8] R. F. Harrington, *Field Computation by Moment Methods*. New York: Wiley–IEEE, 1993.

[9] A. Sommerfeld, *Partial Differential Equations in Physics*. New York: Academic, 1949.

[10] R. Olsen and M. C. Willis, "A comparison of exact and quasi-static methods for evaluating grounding systems at high frequencies," *IEEE Trans. Power Del.*, vol. 11, no. 3, pp. 1071–1081, Jul. 1996.

[11] N. Theethayi, Y. Baba, F. Rachidi, and R. Thottappillil, "On the choice between transmission line equations and full-wave Maxwell's equations for transient analysis of buried wires," *IEEE Trans. Electromagn. Compat.*, vol. 50, no. 2, pp. 347–357, May 2008.

[12] D. Poljak, V. Doric, F. Rachidi, K. Drissi, K. Kerroum, S. V. Tkachenko, and S. Sesnic, "Generalized form of telegrapher's equations for the electromagnetic field coupling to buried wires of finite length," *IEEE Trans. Electromagn. Compat.*, vol. 51, no. 2, pp. 331–337, May 2009.

[13] A. Banos, *Dipole Radiation in the Presence of a Conducting Half-Space*. Oxford: Pergamon, 1966.

[14] A. Banos and J. P. Wesley. (1953, Sep.). The horizontal electric dipole in a conducting half-space. Scripps Inst. Oceanogr., Marine Phys. Lab., Univ. Cal., Rep. 53-33, ch. 4 [Online]. Available: http://repositories.cdlib.org/sio/reference/53-33

[15] J. R. Wait, "Complex resistivity of the earth," in *Progress in Electromagnetic Research,* vol. 1, J. A. Kong, Ed. New York: Elsevier, 1989, pp. 1–175.

[16] T. Takashima, T. Nakae, and R. Ishibashi, "Calculation of complex fields in conducting media," *IEEE Trans. Electr. Insul.*, vol. 15, no. 1, pp. 1–7, Feb. 1980.

[17] J. R. Wait, "The false image of a line current within a conducting half-space," *IEEE Trans. Electromagn. Compat.*, vol. 39, no. 3, pp. 266–267, Aug. 1997.

[18] A. Poggio, R. Bevensee, and E. K. Miller, "Evaluation of some thin wire computer programs," in *Proc. IEEE Antennas Propag. Soc. Int. Symp.*, Jun. 1974, vol. 12, pp. 181–184.