IEEE TRANSACTIONS ON
ELECTROMAGNETIC COMPATIBILITY

A PUBLICATION OF THE IEEE ELECTROMAGNETIC COMPATIBILITY SOCIETY

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NOVEMBER 2011 VOLUME 53 NUMBER 4 IEMCAE (ISSN 0018-9375)

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Digital Object Identifier 10.1109/TEMC.2011.2176159

Per unit capacitance and inductance for cable are 452.3 pF/m and 3.0685 μH/m, respectively. The voltage–current characteristic of the SPD used in the simulation is given in Table I. The 220-V overhead line is assumed to be perpendicular to the 10-kV lines; therefore, the coupling between them can be neglected.

The corresponding worst distance can be obtained by (5): $D_{\mathrm{worst}} = 13.43$ m for 50-m-long overhead line and $D_{\mathrm{worst}} = 20.25$ m for 100-m-long overhead line. The whole system is simulated in EMTDC; the maximum absolute values of voltage on the protected device are recorded for different distances between the SPD and the load and for different loads, as shown in Fig. 3. When the load is resistive, the estimation of formula (5) has a good agreement with the simulation result. While for inductive and capacitive loads, estimation result is prone to be smaller and larger, respectively. This is consistent with the analysis in Section II.C. When the length of overhead line is 50 m and the value of inductance varies from 50 μH to 10 mH, the relative error of estimated D_{worst} varies from 4.2% to 0.5%; and when the value of capacitor varies from 1 pF to 1 nF, the relative error of estimated D_{worst} obtained by (5) varies from 0.2% to 13%, while the relative error of (7) varies from 0.2% to 6%. When the length of overhead line is 100 and 200 m, the maximum relative error of the estimation formula are 9% and 11%, respectively. Therefore, in the design of lightning protection for low-voltage network, it should be avoided that the protected device is separated from the SPD by D_{worst}.

IV. CONCLUSION

It is commonly known that the longer the distance between an SPD and a protected device, the worse the protection effectiveness of the SPD. We have shown in this paper that this is not true for low-voltage power circuits due to the special circuit topology. There exists a worst distance between SPD and protected device. An analytical formula to estimate the worst distance between the protected device and the SPD is presented in this letter when the coupling between 10 kV and 220 V lines is neglected, the results given by this formula has a good agreement with the simulation ones of EMTDC when the length of overhead line is not very long.

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On Inductance of Buried Horizontal Bare Conductors

L. Grcev, B. Markovski, and S. Grceva

***Abstract*—External inductance is one of the basic quantities in the classical approach to the surge and high-frequency analysis of buried horizontal bare conductors. However, there is no consensus, in the modern literature, on the treatment of the effects of the earth surface in the approximate expressions for the inductance, and several different formulas are often used. In this paper, we derive a new expression for the external inductance of buried horizontal conductors that accurately takes into account the effects of the earth surface and compare the errors of the usual approximate formulas. We also propose new approximate formulas that lead to smaller errors for depths of burial less than or equal to 1 m.**

***Index Terms*—Circuit modeling, distributed parameter circuits, grounding electrodes, lightning, transmission line modeling.**

I. INTRODUCTION

High-Frequency and surge modeling of buried horizontal conductors is of interest for a variety of electromagnetic compatibility and lightning-related studies [1]. One of the classical approaches to modeling is a representation of the conductor by a transmission line with uniformly distributed parameters [2]. A simple method to approximately estimate the required unit length parameters was suggested by Sunde (see [2, p. 256]). The leakage conductance, external inductance, and capacitance of the finite-length conductors were first derived from static (dc) conditions. Then, their values were divided by the conductor length, yielding the approximate unit length parameters. This approximate method is still very popular and was recently compared with other methods for high-frequency and surge analysis of grounding electrodes [3].

However, there is no consensus in the modern literature on the treatment of the effects of the earth surface in the approximate expression for the external inductance, and several different formulas (in which the effects of the earth surface are either completely neglected or treated by

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Manuscript received February 24, 2011; revised June 4, 2011; accepted August 1, 2011. Date of publication September 15, 2011; date of current version November 18, 2011. This work was supported in part by the Republic of Macedonia, Ministry of Education and Science and in part by the Macedonian Academy of Sciences and Arts.

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Digital Object Identifier 10.1109/TEMC.2011.2165340

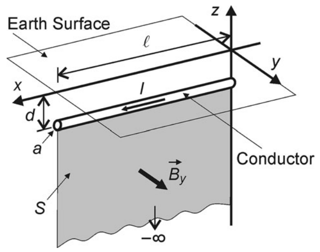

Fig. 1. Geometry of the buried horizontal bare conductor.

image theory) are often used [4]. Nevertheless, an analysis of the error introduced by the different assumptions could not be found in the literature.

In this paper, we derive a new expression for the external inductance of a buried horizontal bare conductor that accurately takes into account the effects of the earth surface. We follow the classical approach described by Sunde [2], but we apply an exact expression for the magnetic field due to a horizontal dc electric dipole in a lossy half-space. We analyze errors of the usual approximate expressions in comparison with the new expression. We also propose new approximate formulas that lead to smaller errors.

II. EXTERNAL INDUCTANCE

Fig. 1 illustrates the geometry of the problem. We consider a horizontal conductor with length ℓ (in meters), radius a (in meters), and depth of burial d (in meters) in uniform earth with conductivity σ (in siemens per meter), permittivity ε (in farads per meter), and permeability μ (in henries per meter). We assume that the longitudinal direct current I (in amperes) is distributed uniformly over the conductor cross section, so it can be approximated by a filament at the conductor axis.

We follow the derivation of the external (or self partial [5]) inductance given in [2]. The longitudinal current is assumed constant along the conductor (see [2, p. 70]). We consider a horizontal dc electric dipole with strength $p = I dx$, located at the conductor axis, i.e., at $(x,\, 0,\, -d)$, where $(0 \le x \le \ell)$, pointing in the positive x-direction (see Fig. 1). Here, we only need the y-component of the magnetic flux density dB_y on the area S (see Fig. 1), i.e., at points with coordinates $(x', 0, z')$. The exact expressions of the magnetic field for a horizontal dc electric dipole were derived by Banos and Wesley [7] considering the static limit ($\omega \to 0$) of the full-wave exact solution involving Sommerfeld integrals (see [8] and [9]), and also considering the independent solution of the dc problem. The derived magnetic field is due to the dipole current and the current distribution set up throughout the conducting media. The required field component in rectangular coordinates (see Fig. 1) is, according to [7], [10], and [11]

$$dB_y = -\frac{\mu I}{4\pi}\left[\frac{z'+d}{R_1^3} + \frac{z'-d}{R_2^3} + \frac{1}{\rho^2}\left(\frac{z'-d}{R_2} + 1\right)\right] dx$$
$$= dB_{y(1)} + dB_{y(2)} + dB_{y(3)} \tag{1}$$

where $\rho = |x' - x|$, $R_1 = \sqrt{\rho^2 + (z'+d)^2}$, and $R_2 = \sqrt{\rho^2 + (z'-d)^2}$.

The first term in the sum on the right side of (1)

$$dB_{y(1)} = -\frac{\mu I}{4\pi}\frac{z'+d}{R_1^3} dx$$

is related to the field due to a dipole in an unbounded uniform medium with the characteristics of the earth, and the second term

$$dB_{y(2)} = -\frac{\mu I}{4\pi}\frac{z'-d}{R_2^3} dx$$

is related to the field due to an image of the dipole in the same unbounded uniform medium elevated at height d above the earth surface. Therefore, the sum of the first and second terms, $dB_{y(1)} + dB_{y(2)}$, is the field as determined by image theory. However, the third term

$$dB_{y(3)} = -\frac{\mu I}{4\pi}\frac{1}{\rho^2}\left(\frac{z'-d}{R_2} + 1\right) dx$$

can be considered to be a "correction" to image theory, which agrees with Wait's arguments [12] that buried horizontal current sources do not have proper images. The sum of the second and third terms on the right side of (1), i.e., $dB_{y(2)} + dB_{y(3)}$, represent the effect of the earth surface.

We determine the self-partial inductance of the conductor L (in henries) by integrating the magnetic flux density through the area S (see Fig. 1) between the conductor surface $(z' = -d - a)$ and infinity $(z' \to -\infty)$ as follows:

$$L = \frac{1}{I}\int_{z'=-\infty}^{-d-a} dz' \int_{x'=0}^{\ell} dx' \int_{x=0}^{\ell} dB_y. \tag{2}$$

(The area S in Fig. 1 corresponds to the area chosen in [1, p. 397] for derivation of the telegrapher's equations.)

We evaluate (2) by considering the three parts of $dB_y(1)$ as follows:

$$L = L_{(1)} + L_{(2)} + L_{(3)}$$
$$= \frac{1}{I}\int_{z'=-\infty}^{-d-a} dz' \int_{x'=0}^{\ell} dx'$$
$$\times \int_{x=0}^{\ell}\left(dB_{y(1)} + dB_{y(2)} + dB_{y(3)}\right). \tag{3}$$

The first part of the sum in (3), $L_{(1)}$, is the inductance in a uniform medium. The exact solution

$$L_{(1)} = \frac{\mu}{2\pi}\ell\left\{\ln\left[\frac{\ell}{a} + \sqrt{\left(\frac{\ell}{a}\right)^2 + 1}\right] - \sqrt{1 + \left(\frac{a}{\ell}\right)^2} + \frac{a}{\ell}\right\} \tag{4}$$

and the usual approximate expression

$$L_{(1)} \cong \frac{\mu}{2\pi}\ell\left[\ln\frac{2\ell}{a} - 1\right], \qquad (\ell \gg a) \tag{5}$$

is well known [5], [6].

Equation (5) is the popular approximate expression for the inductance of buried bare conductors proposed by Sunde (see [2, p. 256]), which neglects the effects of the earth surface.

The solution of the integral for the second part of the sum in (3), $L_{(2)}$, is very similar to (4):

$$L_{(2)} = \frac{\mu}{2\pi}\ell\left\{\ln\left[\frac{\ell}{H} + \sqrt{\left(\frac{\ell}{H}\right)^2 + 1}\right] - \sqrt{1 + \left(\frac{H}{\ell}\right)^2} + \frac{H}{\ell}\right\} \tag{6}$$

where $H = 2d + a$.

If we use the sum $L_{(1)} + L_{(2)}$, we come to another popular approximate expression for the inductance, which is derived from image theory [3]

$$L_{(1)} + L_{(2)} \cong L_{(1)} + \frac{\mu}{2\pi}\ell\left[\ln\frac{2\ell}{H} - 1\right]$$
$$\cong \frac{\mu}{\pi}\ell\left[\ln\frac{2\ell}{\sqrt{2ad}} - 1\right], \quad (\ell \gg a). \tag{7}$$

The details of the derivation of (4) and (6) can be found in [5] and [6]. However, the integral of the third term in (3) is more complex because the integral has singularities at the endpoints $x' = 0$ and $x' = \ell$. Nevertheless, the solution of the integral is straightforward (details of the derivation are given in Appendix A):

$$L_{(3)} = -\frac{\mu}{2\pi} H \times \left\{ \begin{array}{l} \frac{\ell}{H} \ln\left[\frac{\ell}{H} + \sqrt{\left(\frac{\ell}{H}\right)^2 + 1}\right] + 2\left[1 - \sqrt{\left(\frac{\ell}{H}\right)^2 + 1}\right] \\ + \ln\left[\frac{H}{\ell} + \sqrt{\left(\frac{H}{\ell}\right)^2 + 1}\right] - \ln\frac{2H}{\ell} \end{array} \right\}. \tag{8}$$

Finally, the new expression for the self-partial inductance follows from the sum of (4), (6), and (8):

$$L = \frac{\mu}{2\pi}\ell \times \left\{ \begin{array}{l} \ln\left[\frac{\ell}{a} + \sqrt{\left(\frac{\ell}{a}\right)^2 + 1}\right] - \sqrt{1 + \left(\frac{a}{\ell}\right)^2} + \frac{a}{\ell} \\ -\frac{H}{\ell}\left\{ \ln\left[\frac{H}{\ell} + \sqrt{1 + \left(\frac{H}{\ell}\right)^2}\right] - \sqrt{\left(\frac{\ell}{H}\right)^2 + 1} \right. \\ \left. \qquad - \ln\left(\frac{2H}{\ell}\right) + 1 \right\} \end{array} \right\}. \tag{9}$$

Notably, the upper line in (9) is equal to $L_{(1)}$, and the expression in the lower line is related to the effect of the earth surface. The lower line is also related to the error in using the unbounded uniform medium approximation suggested by Sunde (see [2, p. 256]).

In a practical case, when $\ell \gg a$, we have the following approximate expression (details of the derivation are given in Appendix B):

$$L \cong \frac{\mu}{2\pi}\ell \left[\ln\frac{2\ell}{a} + \frac{H}{\ell}\left(\ln\frac{2H}{\ell} - 1\right) - \frac{1}{2}\left(\frac{H}{\ell}\right)^2 + \frac{a}{\ell} - \frac{1}{4}\left(\frac{a}{\ell}\right)^2\right]. \tag{10}$$

In the case where the wire is on the surface of the earth (i.e., $d = 0$), an approximation of the inductance is written as

$$L \cong \frac{\mu}{2\pi}\ell \left[\ln\frac{2\ell}{a} + \frac{a}{\ell}\ln\frac{2a}{\ell} - \frac{3}{4}\left(\frac{a}{\ell}\right)^2\right]. \tag{11}$$

Usually, the conductor is buried at a depth of burial that is much larger than the radius (i.e., $d \gg a$). For such cases, we have the following approximate expression:

$$L \cong \frac{\mu}{2\pi}\ell \left[\ln\frac{2\ell}{a} + \frac{2d}{\ell}\left(\ln\frac{4d}{\ell} - 1\right) - \frac{1}{2}\left(\frac{2d}{\ell}\right)^2\right], \quad (\ell \gg a). \tag{12}$$

We also consider the following simpler variants of (12):

$$L \cong \frac{\mu}{2\pi}\ell \left[\ln\frac{2\ell}{a} + \frac{2d}{\ell}\left(\ln\frac{4d}{\ell} - 1\right)\right], \quad (d \ll \ell, \ell \gg a) \tag{13}$$

and

$$L \cong \frac{\mu}{2\pi}\ell \ln\frac{2\ell}{a}, \quad (d \ll \ell, \ell \gg a). \tag{14}$$

In the next sections, we consider also an additional approximate expression

$$L \cong \frac{\mu}{2\pi}\ell \left[\ln\frac{2\ell}{\sqrt{2ad}} - 1\right] \tag{15}$$

which is not related to the derivation in this paper but is often applied in the modern literature [4].

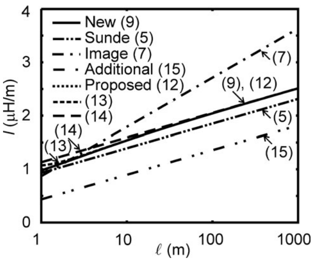

Fig. 2. Dependence of per-unit-length inductance on the length of the conductor (based on different expressions for inductance *L*). Depth of burial: $d = 0.5$ m.

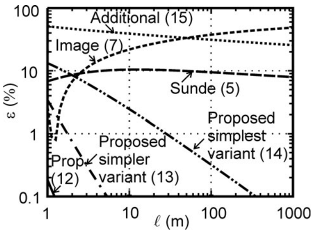

Fig. 3. Errors of the proposed: (12), (13), and (14), and usual: (5), (7), and (15), approximate expressions for self-partial inductance of buried horizontal bare conductors. Depth of burial: $d = 0.3$ m.

III. PER-UNIT-LENGTH INDUCTANCE

The simple method to approximately determine a per-unit-length inductance l (in henries per meter) in [2, p. 256] is by dividing external inductance L by the actual conductor length ℓ:

$$l \cong \frac{L}{\ell}. \tag{16}$$

However, it is clear that (16) may be considered only as a rough estimate of the per-unit-length inductance since the right-hand side still contains the length ℓ of the conductor (see [13, p. 261]). Fig. 2 shows dependence of per-unit-length inductance l (16), for different expressions for external inductance L presented in this paper, on the length ℓ of the conductor. The values of l (16) have larger values for longer conductors and rise to infinity for infinitely long conductor.

One should be careful in using conductor length-dependent per-unit-length parameters in practical high-frequency and surge studies, e.g., of grounding electrodes. Large differences between this method and the more accurate method based on the electromagnetic field theory approach have been reported in [3] especially at high frequencies, for long grounding electrodes, and in very conductive earth, i.e., when inductive effects are dominant. It was reported in [15] that the uniform transmission line approach with conductor length dependent per-unit-length parameters is only valid when the length of the grounding conductor is much smaller than the effective length and the injection current has slow rise times.

IV. COMPARISON OF APPROXIMATE EXPRESSIONS

In this section, we compare different approximate expressions for the inductance L_A with the new solution of integral (2) given in (9) L_E. The error ε (in percent) is evaluated by

$$\varepsilon = \frac{|L_E - L_A|}{L_E} \cdot 100. \tag{17}$$

Figs. 3 and 4 show the error of the following approximate expressions: (5) (proposed by Sunde [2]); (7) (derived from image theory); (15) (additional

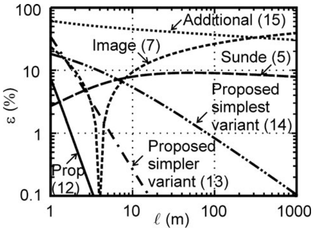

Fig. 4. Errors of the proposed: (12), (13), and (14), and usual: (5), (7), and (15), approximate expressions for self-partial inductance of buried horizontal bare conductors. Depth of burial: $d = 1$ m.

formula); and (12), (13), and (14) (proposed in this paper). The results shown in Figs. 3 and 4 are for depths of burial of $d = 0.3\ \mathrm{m}$ and for $d = 1\ \mathrm{m}$, respectively. Note that the computed errors in Figs. 3 and 4 are identical to the errors of per-unit-length inductances (16).

The results in Figs. 3 and 4 show that the popular formula (5), proposed by Sunde [2], introduces error on the order of several percent for shorter conductors ($\ell < 10\,\mathrm{m}$) to about 10% for longer conductors ($\ell > 10\,\mathrm{m}$) for the depths of burial considered in this paper. Formula (7), derived from image theory, leads to errors larger than 10% for conductors longer than several meters, and the additional formula (15) leads to large errors of several tens of percent for all conductor lengths.

The new approximate expression for inductance proposed in this paper (12), and its simpler forms (13) and (14), in most cases lead to smaller errors. The error depends on the conductor length, with smaller values for longer conductors. For the range of depths of burial considered in this paper, $d \le 1\ \mathrm{m}$, (12) leads to errors smaller than 1% for conductor lengths longer than about twice the depth of burial, $\ell > 2d$, and (13) leads to errors smaller than 1% for about $\ell > 5d$. Even the simplest formula (14) leads to a smaller error than (5) for $\ell > 7d$.

V. CONCLUSION

We have derived a new expression for the external (self-partial) inductance of buried horizontal bare conductor that accurately takes into account the effects of the earth surface. A comparison between the new and currently used approximate expressions shows that the error of the approximate expressions in most cases is near or above 10% for the depths of burial considered in this paper (less or equal to 1 m).

We have also proposed a new approximate formula that is applicable for the same range of depths of burial. The error of the new formula is less than 1% for conductors longer than about twice the depth of burial.

Per-unit-length inductance, determined by dividing the external inductance with the conductor length, may be considered as a rough estimate since its value depends on the conductor length.

APPENDIX A
DETAILS OF DERIVATION OF (8)

We solve the third term of (3) in three steps. The first step is related to the field due to the current *I* in the conductor axis

$$\begin{aligned} B_{y(3)} &= \int_{x=0}^{\ell} dB_{y(3)} \\ &= -\frac{\mu I}{4\pi}\int_{x=0}^{\ell}\left\{\frac{1}{(x'-x)^2}\left[\frac{z'-d}{\sqrt{(x'-x)^2+(z'-d)^2}}+1\right]\right\}dx. \end{aligned}$$

The integral has singularity at $x = x'$

$$\begin{aligned} B_{y(3)} &= \lim_{\delta\to 0}\frac{\mu I}{4\pi} \\ &\times\left\{\begin{array}{c} \displaystyle\int_{x=0}^{x'-\delta}\frac{D}{(x'-x)^2\sqrt{(x'-x)^2+D^2}}dx \\ \displaystyle -\int_{x=0}^{x'-\delta}\frac{1}{(x'-x)^2}dx \\ \displaystyle +\int_{x=x'+\delta}^{\ell}\frac{D}{(x'-x)^2\sqrt{(x'-x)^2+D^2}}dx \\ \displaystyle -\int_{x=x'+\delta}^{\ell}\frac{1}{(x'-x)^2}dx \end{array}\right\} \\ &= \lim_{\delta\to 0}\frac{\mu I}{4\pi}\left[\begin{array}{l} \displaystyle\left[\frac{\sqrt{D^2+(x'-x)^2}}{D(x'-x)}\right]_{x=0}^{x'-\delta}-\left[\frac{1}{x'-x}\right]_{x=0}^{x'-\delta} \\ \displaystyle +\left[\frac{\sqrt{D^2+(x'-x)^2}}{D(x'-x)}\right]_{x=x'+\delta}^{\ell}-\left[\frac{1}{x'-x}\right]_{x=x'+\delta}^{\ell} \end{array}\right] \\ &= \frac{\mu I}{4\pi}\left[\frac{\sqrt{D^2+(x'-\ell)^2}}{D(x'-\ell)}-\frac{1}{x'-\ell}-\frac{\sqrt{D^2+x'^2}}{Dx'}+\frac{1}{x'}\right] \end{aligned}$$

where $D = d - z'$.

The next step is the second integral in (3), which yields the magnetic flux within a horizontal strip of area *S* with length ℓ and infinitesimal width dz'

$$\Delta\Phi_{(3)} = \int_{x'=0}^{\ell} B_{y(3)}\,dx'.$$

The integral has singularities at endpoints $x' = 0$ and $x' = \ell$

$$\begin{aligned} &\Delta\Phi_{(3)} \\ &= \lim_{\delta\to 0}\frac{\mu I}{4\pi}\left\{\begin{array}{l} \displaystyle\int_{x'=0}^{\ell-\delta}\frac{\sqrt{D^2+(x'-\ell)^2}}{D(x'-\ell)}dx'-\int_{x'=0}^{\ell-\delta}\frac{1}{(x'-\ell)}dx' \\ \displaystyle -\int_{x'=\delta}^{\ell}\frac{\sqrt{D^2+x'^2}}{Dx'}dx'+\int_{x'=\delta}^{\ell}\frac{1}{x'}dx' \end{array}\right\} \\ &= \lim_{\delta\to 0}\frac{\mu I}{4\pi}\left\{\begin{array}{c} \displaystyle\left[\frac{\sqrt{D^2+u^2}}{D}-\ln\frac{D+\sqrt{D^2+u^2}}{u}\right]_{u=-\ell}^{-\delta} \\ \displaystyle -\left[\ln|u|\right]_{u=-\ell}^{-\delta} \\ \displaystyle -\left[\frac{\sqrt{D^2+x'^2}}{D}-\ln\frac{D+\sqrt{D^2+x'^2}}{x'}\right]_{x'=\delta}^{\ell} \\ \displaystyle +\left[\ln|x'|\right]_{x'=\delta}^{\ell} \end{array}\right\} \\ &= \frac{\mu I}{2\pi}\left\{1-\ln[2D]-\frac{\sqrt{D^2+\ell^2}}{D}+\ln\left[D+\sqrt{D^2+\ell^2}\right]\right\}. \end{aligned}$$

Here, we use a change in variables $u = x' - \ell,\ du = dx'$, and the integral (13) of Paul (see [5, p. 370]) (also 241.01 of Dwight [14])

$$\int\frac{\sqrt{x^2+a^2}}{x}dx = \sqrt{x^2+a^2}-a\ln\frac{a+\sqrt{x^2+a^2}}{x}.$$

Finally, the total magnetic flux due to $B_{y(3)}$ is written as

$$\Phi_{(3)} = \int_{-\infty}^{-d-a}\Delta\Phi_{y(3)}\,dz' = \int_{D=2d+a}^{\infty}\Delta\Phi_{y(3)}\,dD$$

$$= \lim_{\delta\to\infty} \frac{\mu I}{2\pi} \int_{D=2d+a}^{\delta}$$

$$\times \left\{ 1 - \ln\left[2D\right] - \frac{\sqrt{D^2+\ell^2}}{D} + \ln\left[D + \sqrt{D^2+\ell^2}\right] \right\} dD$$

$$= \lim_{\delta\to\infty} \frac{\mu I}{2\pi}\ell \left\{ \begin{array}{l} \ln\left[\frac{\ell}{D} + \sqrt{1+\left(\frac{\ell}{D}\right)^2}\right] \\ \qquad\qquad + \frac{D}{\ell}\ln\left[1+\sqrt{1+\left(\frac{\ell}{D}\right)^2}\right] \\ +2\frac{D}{\ell}\left[1-\sqrt{1+\left(\frac{\ell}{D}\right)^2}\right] - \frac{D}{\ell}\ln 2 \end{array} \right\} \Bigg|_{D=2d+a}^{\delta}$$

from which we derive (8).

APPENDIX B
DETAILS OF DERIVATION OF (10)

We use approximate expressions 602.1 from Dwight [14] and (16) from Paul (see [5, p. 369]), respectively,

$$\ln\left[\frac{m}{n} + \sqrt{\left(\frac{m}{n}\right)^2 + 1}\right] = \ln\frac{2m}{n} + \frac{1}{4}\left(\frac{n}{m}\right)^2$$

$$-\frac{3}{32}\left(\frac{n}{m}\right)^4 + \cdots, \qquad \frac{m}{n} > 1$$

$$= \frac{m}{n} - \frac{1}{6}\left(\frac{m}{n}\right)^3 + \frac{3}{40}\left(\frac{m}{n}\right)^5 - \cdots, \qquad \frac{m}{n} < 1$$

$$\sqrt{1+\left(\frac{n}{m}\right)^2} = 1 + \frac{1}{2}\left(\frac{n}{m}\right)^2 - \frac{1}{8}\left(\frac{n}{m}\right)^4$$

$$+\frac{1}{16}\left(\frac{n}{m}\right)^6 - \cdots, \qquad \frac{n}{m} \le 1$$

$$= \frac{n}{m} + \frac{1}{2}\frac{m}{n} - \frac{1}{8}\left(\frac{m}{n}\right)^3 + \frac{1}{16}\left(\frac{m}{n}\right)^5 - \cdots, \qquad \frac{n}{m} \ge 1$$

in (9), obtaining

$$L = \frac{\mu}{2\pi}\ell$$

$$\times \left\{ \begin{array}{l} \left[\ln\frac{2\ell}{a} + \frac{1}{4}\left(\frac{a}{\ell}\right)^2 - \cdots\right] - \left[1 + \frac{1}{2}\left(\frac{a}{\ell}\right)^2 - \cdots\right] + \frac{a}{\ell} \\ -\frac{H}{\ell}\left\{\left[\frac{H}{\ell} - \frac{1}{6}\left(\frac{H}{\ell}\right)^3 + \cdots\right] - \left[\frac{\ell}{H} + \frac{1}{2}\frac{H}{\ell} - \cdots\right]\right. \\ \qquad\qquad\qquad \left. - \ln\frac{2H}{\ell} + 1\right\} \end{array} \right\}$$

from which we derive (10).

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Improvements to a Method for Estimating the Maximum Radiated Emissions From PCBs With Cables

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***Abstract*—It has been shown in previous studies that the coupling from ICs, traces, or heatsinks on a printed circuit board to an attached cable can be modeled by placing equivalent common-mode sources between the board and the cable. In a 2008 paper published in the IEEE TRANSACTIONS ON ELECTROMAGNETIC COMPATIBILITY, a closed-form expression was developed to estimate the maximum radiated emissions from board-source-cable structures. While this expression is reasonably accurate for frequencies not exceeding 500 MHz, it may unnecessarily overestimate the emissions in some situations, especially when the maximum frequency of interest is extended beyond 500 MHz. This paper introduces two enhancements to the previously introduced closed-form expression based on improved methods for determining the maximum value of $F(\theta,k,l_{\mathrm{ant}})$ and the effective board size. The new closed-form expression is evaluated for various board geometries and frequency ranges by comparing the estimated maximum radiated emissions to full-wave simulation results.**

***Index Terms*—Common mode, electromagnetic modeling, electromagnetic radiation, imbalance difference model.**

Manuscript received May 15, 2011; revised July 9, 2011; accepted August 8, 2011. Date of publication September 26, 2011; date of current version November 18, 2011. This work was supported in part by the National Science Foundation I/UCRC for Electromagnetic Compatibility.

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Digital Object Identifier 10.1109/TEMC.2011.2165217