COMPARISON OF ELECTROMAGNETIC MODELS OF LIGHTNING RETURN STROKES USING CURRENT AND VOLTAGE SOURCES

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ABSTRACT: Electromagnetic models based on antenna theory have been recently applied in lightning studies. Their practical use is facilitated by existing thin-wire antenna numerical codes that utilize localized voltage sources. Such voltage sources have to be tuned to provide the lightning channel-base current for which experimental data are available. This involves some preparatory steps, such as the determination of the input impedance of the antenna (in the frequency domain) and Fourier and inverse Fourier transforms. These steps may involve methodological and computational errors. In this paper, we present initial results of an effort to introduce an alternative current source model formulation that will enable a direct specification of the channel-base current in electromagnetic models of lightning.

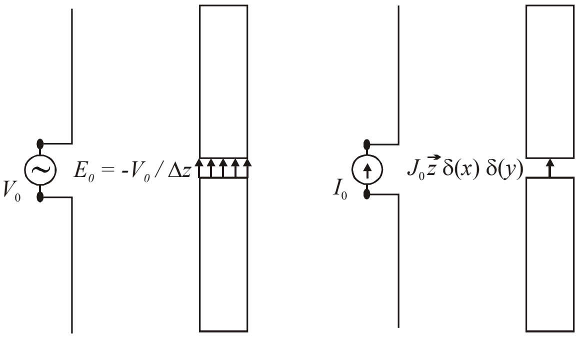
INTRODUCTION

One of the classes of models applied recently in lightning studies is the so-called electromagnetic models [*Rakov and Uman*, 1998]. They are usually based on thin-wire antenna approximation and involve a numerical solution of Maxwell's equations using the method of moments [*Harrington*, 1993] to find the current distribution along the lightning channel, from which electromagnetic fields can be computed. *Moini et al.* [2000] and *Kordi et al.* [2002] applied the time-domain approach utilizing the Thin-Wire Time-Domain code (TWTD) [e.g., *van Blaricum*, 1972], and *Baba and Ishii* [2001, 2003] applied the frequency-domain approach utilizing the Numerical Electromagnetic Code (NEC-2) [*Burke*, 1980].

One potential problem in application of existing antenna computer codes in lightning studies is related to the use of a localized voltage source model. Since lightning is usually characterized by measured or analytically specified currents at the channel base, the natural choice in modeling would be a current instead of voltage source. However, current sources are usually not provided in the existing antenna codes. This short paper explores the possibility of using current sources in electromagnetic models of lightning. We will confine our analysis to the frequency domain approach and the implementation to one well-known thin-wire, moment-method antenna code by *Richmond* [1974].

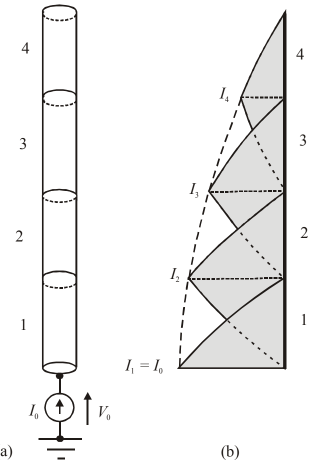
MODELING OF SOURCES IN ANTENNA THEORY

As an example Fig. 1 illustrates two alternative possibilities for energizing the antenna by a voltage and a current source and their possible field representations. *Jordan and Balmain* [1968] discusses other different source model representations. The impressed electric field E_0 (Fig. 1a) and Dirac delta current source J_0 (Fig. 1b) are equivalent to localized voltage and current sources, respectively, applied across an infinitesimally narrow gap (delta gap sources).



(a) (b)

Fig. 1. Circuit and field representations of an antenna fed by (a) voltage and (b) current source.



(a) (b)

Fig. 2. (a) Antenna represented by four segments and (b) approximation of current distribution along the antenna.

Figure 2 illustrates the implementation of the voltage or current source in a thin-wire, moment-method solution. We consider the piecewise sinusoidal Galerkin method [*Richmond* 1992]. The antenna is divided in fictitious segments, and the current along the antenna is represented by samples at the segments' junction points. Current sample I_k at each junction point determines the distributions of current along the neighboring segments, which are approximated by sinusoidal functions along segments with the assumption of zero current at the upper ends. Such current distributions at any two neighboring segments form the so-called 'sinusoidal dipoles'. The overall current distribution along the antenna is approximated by a superposition of overlapped 'sinusoidal dipoles'. The longitudinal current $I(\ell)$ at point ℓ is then approximated along the conductors' network by a linear combination of M expansion functions $F_k(\ell)$:

$$I(\ell)=\sum_{k=1}^{M} I_k F_k(\ell), \quad F_k(\ell)=\frac{\sinh\left[\gamma(\ell-\ell_k)\right]}{\sinh(\gamma d)}, \quad \gamma^2=-\omega^2\mu_0\varepsilon_0 \tag{1}$$

Here d denotes the length of the segment, ℓ_k denotes the junction point, the constants of permittivity and permeability of the air are denoted ε_0 and μ_0, respectively, $j=\sqrt{-1}$, and ω is angular frequency. It should be noted that the time-variation $\exp(j\omega t)$ is suppressed.

We consider a straight, vertical thin antenna over a perfectly conducting earth with a source connected between the base of the antenna and the earth (Fig. 2). The current at the antenna base is represented by an additional 'sinusoidal monopole', a current distribution defined by a current sample at the base and sinusoidal distribution along the first segment. The unknown current samples along the antenna may be evaluated applying the well-known moment methods [*Richmond* 1992]. For the four-segment example in Fig. 2, if the voltage source V_0 or current source I_0 is used at the base, the following matrix equations lead to the solution for the current distribution I_n.

$$\text{Voltage source:} \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} \\ z_{21} & z_{22} & z_{23} & z_{24} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ z_{41} & z_{42} & z_{43} & z_{44} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} V_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad \text{Current source:} \begin{bmatrix} 1 & 0 & 0 & 0 \\ z_{21} & z_{22} & z_{23} & z_{24} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ z_{41} & z_{42} & z_{43} & z_{44} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} I_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

Elements z_{mn} of the matrix are related to mutual electromagnetic interactions between segments m and n. It is worth noting that elements z_{mn} of the two matrices in (2) are equal except for the first row. Details of the implementation are available elsewhere [*Grcev* 1992, 1996].

ANTENNA MODEL OF LIGHTNING CHANNEL IN FREQUENCY DOMAIN

Similarly to *Moini et al.* [2000], the lightning channel is modeled as a 7500-m high vertical antenna with a diameter of 10 cm. To realize a slower propagation velocity of the current wave in the channel, $0.43\cdot c$, in accordance with measurements, the relative permittivity of the surrounding medium is changed from $\varepsilon_r = 1$ to $\varepsilon_r = 5.3$ [*Moini et al.* 2000]. Once the current distribution is determined, the electromagnetic fields are computed in the medium with relative permittivity of air $\varepsilon_r = 1$. As an alternative, we performed computations of the currents for the medium with relative permittivity of air $\varepsilon_r = 1$, similarly to *Kordi et al.* [2002]. In the preliminary analysis reported in this paper, the ohmic losses in the antenna are neglected. We used for computations the channel-base current with a peak value of about 11 kA and a peak current rate of rise of about 105 kA/μs [*Nucci et al.* 1990].

For the transfer of the results into the time domain by discrete Fourier techniques, a frequency range up to $F_m = 4.1$ MHz and frequency samples at $\Delta f = 1$ kHz were required for reasonable convergence of the results. This corresponds to 8192 time samples at $\Delta t = 1 / 2F_m = 0.12$ μs. The maximum frequency corresponds to a 30 m wavelength; therefore, the segment length was chosen to be at least 3 m. The solution by moment method with 2500 segments for 4100 frequencies requires about 500 hours on a 1.6 GHz Pentium 4 PC. Limited computer resources required a compromise with the accuracy in this study: 3 m segment lengths were applied only up to 1 km, and longer segments were used for the higher parts of the antenna.

As an example of the complexity of the current distribution in frequency domain, Figs. 3 and 4 show the current distribution along the antenna (segmented along the whole channel length at 3 m) at 1 MHz[1]. Figure 5 shows current at 1 km as a function of frequency (computed with different segment lengths along the channel). Both cases correspond to a harmonic current source at the base with a constant 1 A amplitude in the whole spectrum. Figure 6 shows the voltage of the equivalent voltage source that would produce the same harmonic current at the base with 1 A amplitude in the whole spectrum and consequently the same current distribution along the antenna.

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[1] All results are for the current distribution determined using relative permittivity $\varepsilon_r = 5.3$, unless stated otherwise.

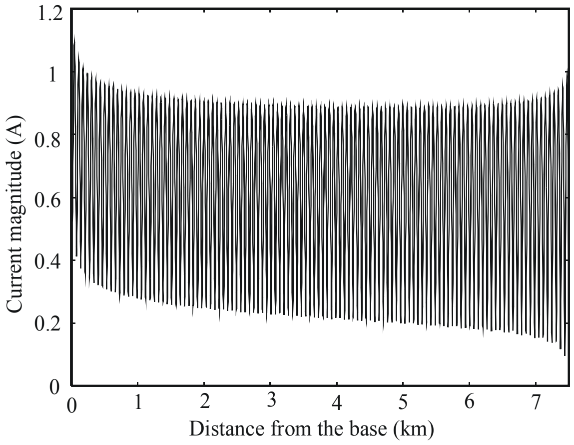

Fig. 3. Current along antenna at 1 MHz (magnitude)

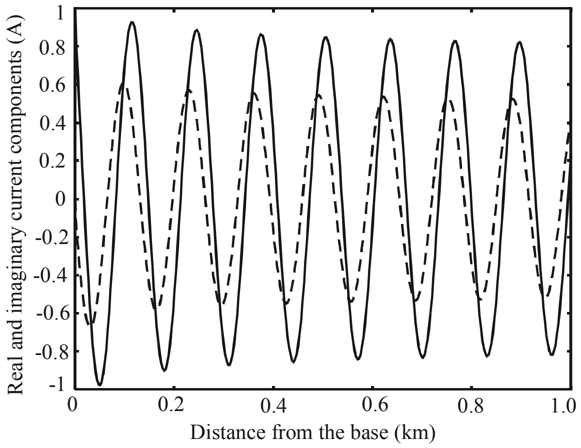

Fig. 4. Current along antenna at 1 MHz for the bottom 1 km (solid line – real component, broken line – imaginary component)

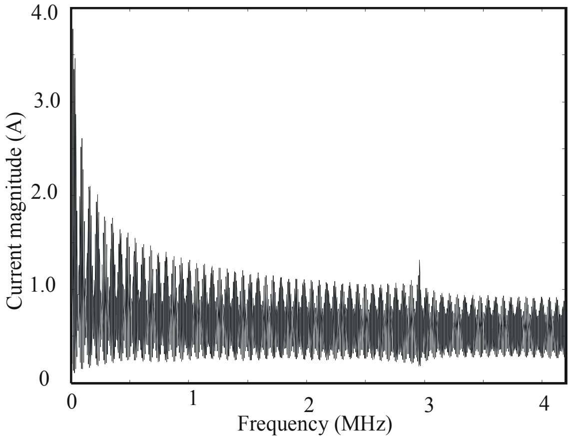

Fig. 5. Current at 1 km above ground

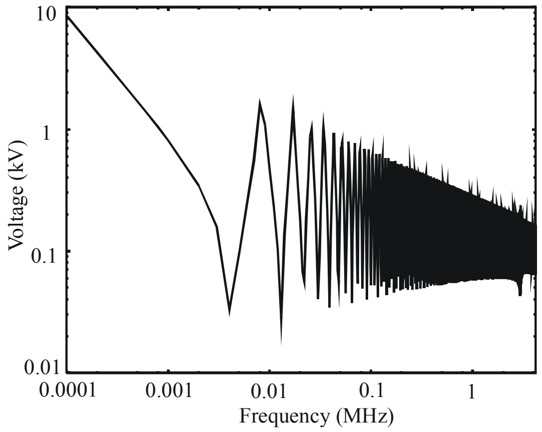

Fig. 6. Voltage of equivalent voltage source at the antenna base

RESULTS IN TIME DOMAIN

To illustrate the capabilities of the method presented in this paper several results in the time domain are given bellow for the channel-base current waveform used by *Nucci et al.* [1990]. Figure 7 shows current waveforms at the channel base and at 1 km above ground for the two cases: for modified medium with relative permittivity $\varepsilon_r = 5.3$, and for medium with relative permittivity $\varepsilon_r = 1$. It can be seen that the numerical results reproduce the expected attenuation and dispersion of the current wave as it propagates along the antenna. Figures 8 and 9 illustrate the vertical electric field at the earth's surface at 500 m and 5 km, respectively. The results are generally comparable to results obtained using other electromagnetic models [e.g., *Moini et al.* 2000; *Kordi et al.* 2002]. However, more work is necessary for the detailed validation of the method presented in this paper. In particular, the influence of the parameters of the method of moments, such as segmentation of the antenna, and the influence of parameters of Discrete Fourier transform, related to sampling and truncation of functions in time and frequency domains, have to be carefully examined.

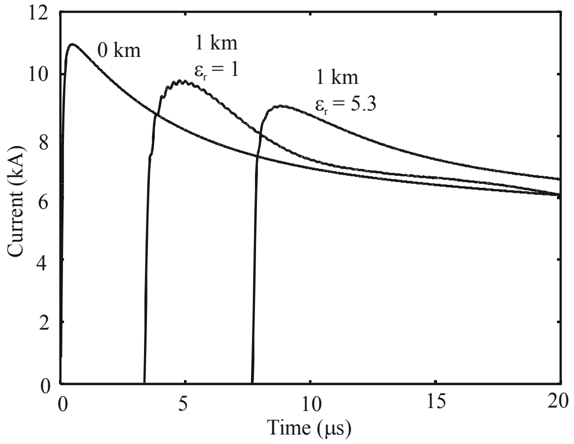

Fig. 7. Current waveforms at the channel base (0 km) and at 1 km above ground.

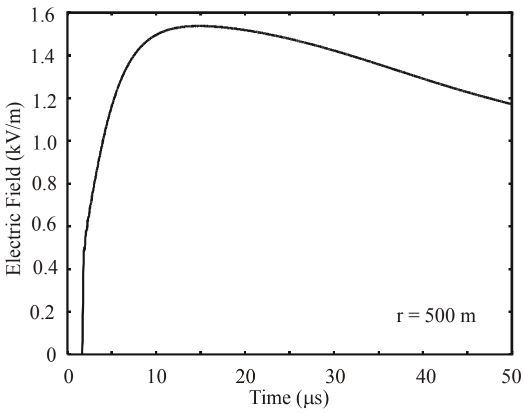

Fig. 8. Vertical electric field at 500 m.

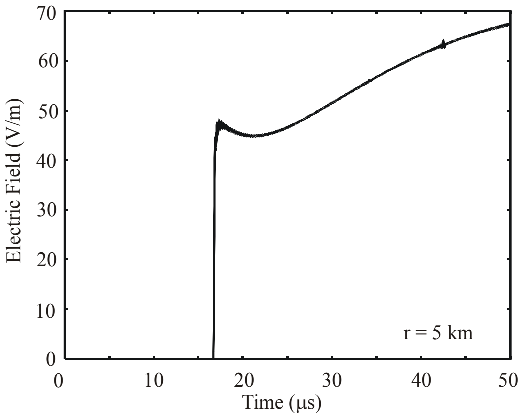

Fig. 9. Vertical electric field at 5 km.

CONCLUSIONS

Practical use of electromagnetic models of the lightning return stroke is facilitated by existing antenna theory techniques, in both time and frequency domains, that involve a voltage source for energizing the antenna. Since channel-base current is usually known rather than voltage, tuning of the voltage source is necessary to provide the specified current.

This paper describes a simple modification that introduces the current source, which enables direct specification of the channel-base current for energizing the antenna in the frequency domain. This model offers new insights into the phenomena involved and may have potential for testing the validity of other models. The model is illustrated by numerical examples.

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